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Thermo-diffusion and Diffusion-thermo Effects on Free Convection Heat and Mass Transfer from Vertical Surface in a Porous Medium with Viscous Dissipation in the Presence of Thermal Radiation

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

The finite difference method is employed to examine free convective heat and mass transfer in a steady two-dimensional fluid flow from vertical surface in porous medium. In this study thermal radiation, viscous dissipation and Soret, Dufour effects are taken into consideration. The two-dimensional boundary-layer governing partial differential equations have been transformed by a similarity transformation into a system of ordinary differential equations which are solved numerically. The effects of different involved parameters such as Lewis number, Soret number, thermal radiation and viscous dissipation parameter on velocity, temperature and concentration profiles are plotted and discussed in the paper.

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Keywords: Free convection; porous medium; viscous dissipation; Dufour; Soret; thermal radiation; finite difference methods.

NOMENCLATURES

S_r - *Soret number*
T - *Temperature*
x, y - *Cartesian co-ordinates along and normal to the surface, respectively*
α_m - *Thermal diffusivity*
C - *Concentration*
C_p - *Specific heat at constant pressure*
C_s - *Concentration susceptibility*
Ra_x - *Local Rayleigh number*
u, v - *Darcian velocities in the x and y directions*
θ - *Dimensionless temperature*
ρ - *Density*
ψ - *Stream function*
β_T - *Coefficient of thermal expansion*
β_C - *Coefficient of concentration expansion*
ϕ - *Dimensionless concentration*
η - *Similarity variable*
υ - *Kinematic viscosity*
D_m - *Mass diffusivity*
f - *Dimensionless stream function*
K - *Darcy permeability*
k_T - *Thermal diffusion ratio*
α^* - *Mean absorption coefficient*
R - *Thermal radiation parameter*

1. INTRODUCTION

During the past several decades, convective flow through porous media has been a subject of considerable research interest of a large number of scholars due to its diverse engineering applications. These applications include, but are not limited to, for example heat exchangers in high heat flux applications such as electronic equipment, insulation of the heated body, thermal energy storage and sensible heat storage beds, drying process (wood and food products), air conditioning and filtration process. When heat and mass transfer occur simultaneously in a moving fluid, the relation between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradient and this is the Soret or thermal–diffusion effect. These effects are considered as second-order phenomena, on the basis that they are of smaller order of magnitude than the effects described by Fourier's and Fick's laws, but they may become significant in areas such geosciences or hydrology. Steady and transient free convection coupled heat and mass transfer by natural convection in a fluid-saturated porous medium has attracted considerable attention in the last years, due to many important engineering and geophysical applications. Recent books by [1,2,3] present a comprehensive account of the available information in the field.

The effects of wall shear stress on unsteady Magnetohydrodynamic conjugate flow in a porous medium with ramped wall temperature studied by [4]. Unsteady Magnetohydrodynamic free convection flow in a porous medium with constant mass diffusion and Newtonian heating studied by [5]. Unsteady Magnetohydrodynamic free convection flow of a second grade fluid in a porous medium with ramped wall temperature discussed by [6]. Heat transfer analysis of Magnetohydrodynamic thin film flow of an unsteady second grade fluid past a vertical oscillating belt studied by [7]. Magnetohydrodynamic flow of a nanofluid over a nonlinear stretching sheet discussed by [8]. The exact solutions for free convection flow of nanofluids with ramped wall temperature observed by [9]. Conjugate effects of heat and mass transfer on Magnetohydrodynamic free convection flow over an inclined plate embedded in a porous medium was studied [10]. Unsteady Magnetohydrodynamic oscillatory flow of viscoelastic fluids in a porous channel with heat and mass transfer studied by [11].

Effect of doubly stratification on free convection in darcian porous medium has been studied by [12]. Recently [13], investigated the effects of Soret and Dufour on free convection heat and mass transfer from a vertical surface in a doubly stratified darcy porous medium. They have neglected effect of Magnetohydrodynamics (Viscous dissipation). The effect of magnetic field and double dispersion on free convection heat and mass transport considering the Soret and Dufour effects in a non – darcy porous medium studied by [14]. The effect of variable thermal conductivity and heat source/sink on Magnetohydrodynamic flow near a stagnation point on a linearly stretching sheet observed by [15]. The heat and mass transfer characteristics of natural convection about a vertical surface embedded in a saturated porous medium subjected to a chemical reaction by taking account the Dufour and Soret effects analyzed by [16]. Soret and Dufour effect on steady Magnetohydrodynamic free convection flow past a semi infinite moving vertical plate in a porous medium with viscous dissipation studied by [17]. The effects of thermal radiation and heat transfer over an unsteady stretching surface embedded in a porous medium in the presence of heat source or sink studied by [18]. Finite difference analysis of radiative free convection flow past an impulsively started vertical plate with variable heat and mass flux discussed by [19]. The hydromagnetic flow of a viscous incompressible fluid past an oscillating vertical plate embedded in a porous medium, radiation with viscous dissipation and variable heat and mass diffusion studied by [20]. Free convective flow of visco-elastic fluid in a vertical channel with Dufour effect studied by [21]. Free convective fluctuating Magnetohydrodynamic flow through porous media past a vertical porous plate with variable temperature and heat source analyzed by [22]. Chemical reaction effect on Magnetohydrodynamic free convective surface over a moving vertical plane through porous medium discussed by [23].

The objective of this paper is to study simultaneous heat and mass transfer by natural convection from a vertical surface embedded in a fluid saturated darcian porous medium including Soret, Dufour effects with viscous dissipation effect in the presence of thermal radiation.

2. MATHEMATICAL ANALYSIS

Consider the natural convection in a porous medium saturated with a Newtonian fluid on a vertical flat plate. The x-coordinate is measured along the surface and the y-coordinate normal to it (see Fig. 1). The temperature of the ambient medium is T_∞ and the wall temperature is T_w. The flow along the vertical flat plate contains a species A slightly soluble in the fluid B, the concentration at the plate surface is C_w and the solubility of A in B far away from the plate is C_∞ .

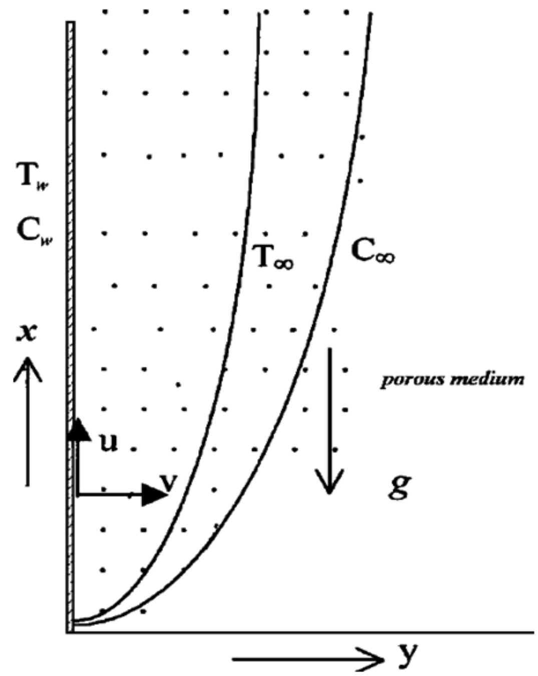

Fig. 1. Flow model and physical coordinate system

Several assumptions are used throughout the present paper: (a) the fluid and the porous medium are in local thermodynamic equilibrium; (b) the flow is laminar, steady state and two-dimensional; (c) the porous medium is isotropic and homogeneous; (d) the properties of the fluid and porous medium are constant; (e) the Boussinesq approximation is valid and the boundary layer approximation is applicable; (f) the concentration of dissolved A is small enough. In-line with these assumptions, the governing equations describing the conservation of mass, momentum, energy and concentration can be written as follows, [24].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=\frac{gK}{\upsilon}\left[\beta_T(T-T_\infty)+\beta_C(C-C_\infty)\right] \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{D_m}{C_s}\frac{k_T}{C_p}\frac{\partial^2 C}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \quad (4)$$

Where all quantities are defined in the nomenclature.

The boundary conditions of the problem are

$$\begin{aligned} &y=0: v=0, T=T_w, C=C_w \\ &y\to\infty: u\to 0, T\to T_\infty, C\to C_\infty \end{aligned} \quad (5)$$

Where T_w, T_∞, C_w and C_∞ have constant values.

Using the Rosseland approximation for radiation [25], radiative heat flux is simplified as:

$$q_r=-\frac{4\sigma}{3\alpha^*}\frac{\partial T^4}{\partial y} \quad (6a)$$

Where σ and α^* are the Stefan-Boltzman constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow are such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms we get:

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \quad (6b)$$

Using equations (6a) and (6b) equation (3) becomes:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{D_m}{C_s}\frac{k_T}{C_p}\frac{\partial^2 C}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{16\sigma T_\infty^3}{3\alpha^*\rho C_p}\frac{\partial^2 T}{\partial y^2} \quad (7)$$

Equations (1), (2), (4), (5), (7) are now nondimensionalized using the following quantities:

$$\psi=\alpha_m Ra_x^{1/2} f(\eta),$$

$$\theta=(T-T_\infty)/(T_w-T_\infty),$$

$$\phi=(C-C_\infty)/(C_w-C_\infty),\ \eta=\frac{y}{x}Ra_x^{1/2}, \quad (8)$$

Where the stream function ψ is defined in the usual way

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x} \quad (9)$$

And $Ra_x=gK\beta(T_w-T_\infty)x/(\upsilon\alpha_m)$ is the local Rayleigh number.

The governing equations become

$$f'=\theta+N\phi \quad (10)$$

$$\left(1+\frac{16}{3R}\right)\theta''-f\theta'+D_f\phi''+E_c(f'')^2=0 \quad (11)$$

$$\frac{1}{Le}\phi''+f\phi'+S_r\theta''=0 \quad (12)$$

Where Le, D_f, S_r, N and R are Lewis, Dufour, Soret, Sustentation and Radiation parameter respectively.

$$Le=\frac{\alpha_m}{D_m}, D_f=\frac{D_m k_T(C_w-C_\infty)}{C_s C_p\alpha_m(T_w-T_\infty)},$$

$$S_r=\frac{D_m k_T(T_w-T_\infty)}{C_s C_p\alpha_m(C_w-C_\infty)},$$

$$N=\frac{\beta_c(C_w-C_\infty)}{\beta_T(T_w-T_\infty)}$$

$$R=\frac{\rho C_p\alpha_m\alpha^*}{\sigma T_\infty^3} \quad (13)$$

We notice that N is positive for thermally assisting flows, negative for thermally opposing flows and zero for thermal-driven flows.

The transformed boundary conditions are

$$f(0)=0, \theta(0)=1, \phi(0)=1$$

$$\theta \to 0, \phi \to 0 \text{ as } y \to \infty \quad (14)$$

The parameters of engineering interest for the present problem are the local Nusselt number and local Sherwood number, which are given by the expressions

$$Nu_x / Ra_x^{1/2} = -\theta'(0),\ Sh_x / Ra_x^{1/2} = -\phi'(0). \quad (15)$$

3. MATHEMATICAL SOLUTION

The set of non-linear ordinary differential equations (10) - (12) must be solved along with the boundary conditions (14). Since analytical solutions do not exist. One has to use numerical techniques. In this paper, the equations have been solved numerically by using Crank Nicolson implicit finite difference method. The implicit finite difference scheme is used to solve the equations (10), (11) and (12). We get the following equations.

$$f[i] = a_1[i] f[i+1] - d_1[i] \quad (16)$$

$$\theta[i] = a_2[i]\theta[i+1] + b_2[i]\theta[i-1] + d_2[i] \quad (17)$$

$$\phi[\mathrm{i}] = a_3[i]\phi[i+1] + b_3[i]\phi[i-1] + d_3[i] \quad (18)$$

Where

$$a_1[i] = 1$$

$$d_1[i] = h*\theta[i] + h*N*\phi[i]$$

$$a_2[i] = \frac{1}{2} - \frac{3*R*h}{12*R+64} f[i]$$

$$b_2[i] = \frac{1}{2} + \frac{3*R*h}{12*R+64} f[i]$$

$$\mathrm{d}_2[i] = \frac{3*R*h^2}{6*R+32}\left[D_f*\phi''[i] + E_c*(f''[i])^2\right]$$

$$a_3[i] = \frac{Le*f[i]*h}{4} + \frac{1}{2}$$

$$b_3[i] = \frac{1}{2} - \frac{Le*f[i]*h}{4}$$

$$d_3[i] = \frac{Le*S_r*h^2}{2}\theta''[i]$$

To obtain the numerical solutions the equations (10), (11) and (12) with boundary conditions (14) are solved by using Gauss-Seidel iterative method. A step size of $\Delta\eta$ = 0.01 was selected to be satisfactory for a convergence criteria of 10^{-5} in all cases. The value of η_∞ was found to each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. In order to see the effect of step size $\Delta\eta$ we ran the code for our model with two different step sizes $\Delta\eta$ = 0.01, $\Delta\eta$ = 0.001 and each case we found very good agreement between them.

4. RESULTS AND DISCUSSION

As a result of the numerical calculations, the dimensionless velocity, temperature, and concentration profiles for the flow under consideration are obtained and their behavior has been discussed for variations in the governing parameters, namely, the Dufour parameter D_f, Soret parameter S_r, Lewis number *Le,* Radiation parameter *R*, Eckert parameter E_c and *N*.

The velocity profiles are presented from Fig.2 to Fig. 5. From Fig. 2, we observed that as the buoyancy ratio *N* increases, the velocity profiles increases. Lewis number (*Le*) is defined as the ratio of thermal diffusivity to mass diffusivity, as the Lewis number increases, the velocity increases in Fig. 3. The influence of viscous dissipation parameter E_c is observed in Fig. 4. The Eckert number (E_c) expresses the relationship between the kinetic energy of the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stress. It is observed that the greater viscous dissipative heat causes an increase in the velocity profiles across the boundary layer. Fig. 5 depicts the effect of varying thermal radiation parameter R on the flow velocity. We observed that the thermal radiation enhances convection flow.

The temperature profiles are plotted from Fig. 6 to Fig. 8. The influence of Soret number (S_r) is presented in Fig. 6. The Soret number defines the effect of the temperature gradients inducing significant mass diffusion effects. As the Soret number increases, the temperature profiles increases. The temperature profiles decreases with increase of Viscous dissipation parameter (E_c) in Fig. 7. The influence of radiation parameter *R* is depicted in Fig. 8. The effect of radiation is to decrease the rate of energy transport to the fluid, there by decreasing the temperature of the fluid.

The concentration profiles are plotted from Fig.9 to Fig. 10. It is seen that the thickness of the concentration boundary layer decreases as the Lewis number (Le) increases in Fig. 9. In Fig. 10, we observed that as the viscous dissipation increases, the concentration of boundary layer decreases.

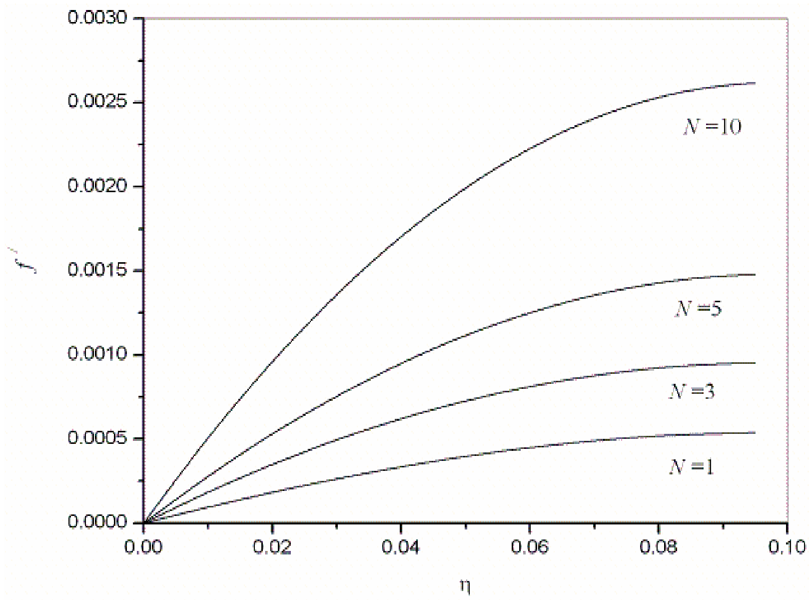

Fig. 2. Velocity profiles for different values of N **with** $S_r = 0.001, D_f = 10.0$

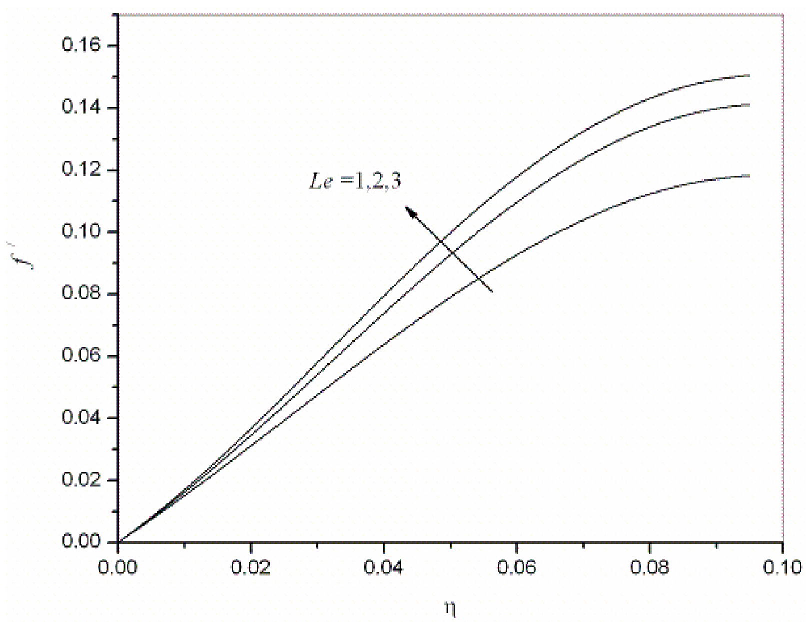

Fig. 3. Velocity profiles for different Le **with** $S_r = 0.001, D_f = 10.0, N = 1$

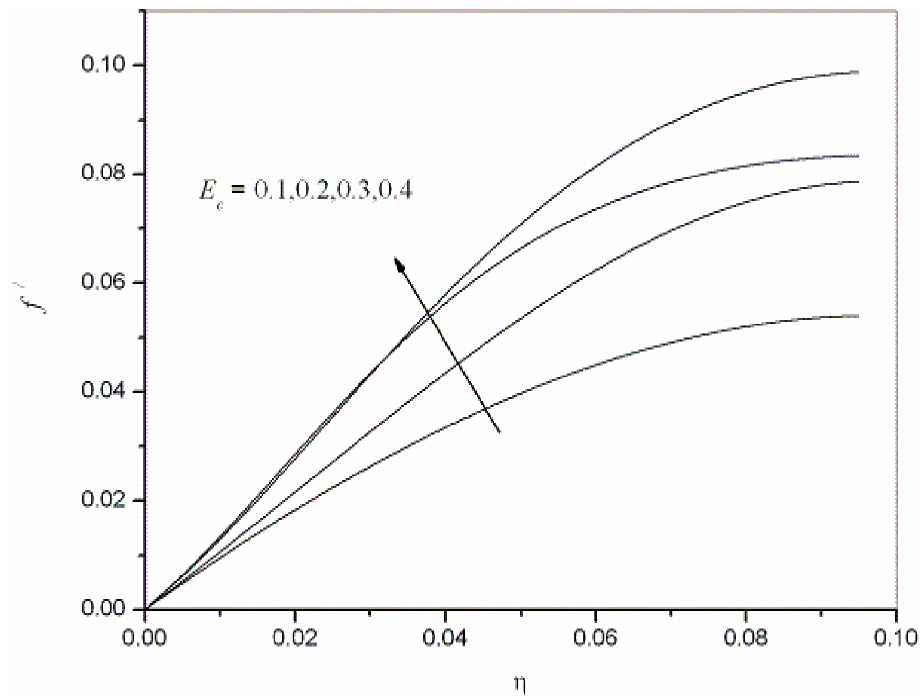

Fig. 4. Velocity profiles for different Eckert parameters E_c with $S_r = 0.001, D_f = 10.0$

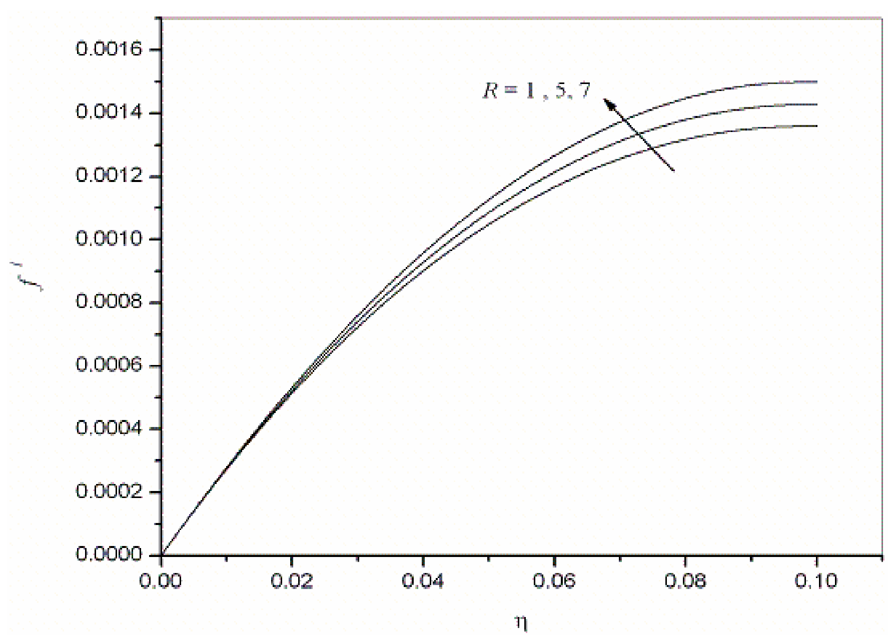

Fig. 5. Velocity profiles for different values of *R* with
$S_r = 0.001, D_f = 10.0, N = 1, E_c = 0.5, Le = 2$

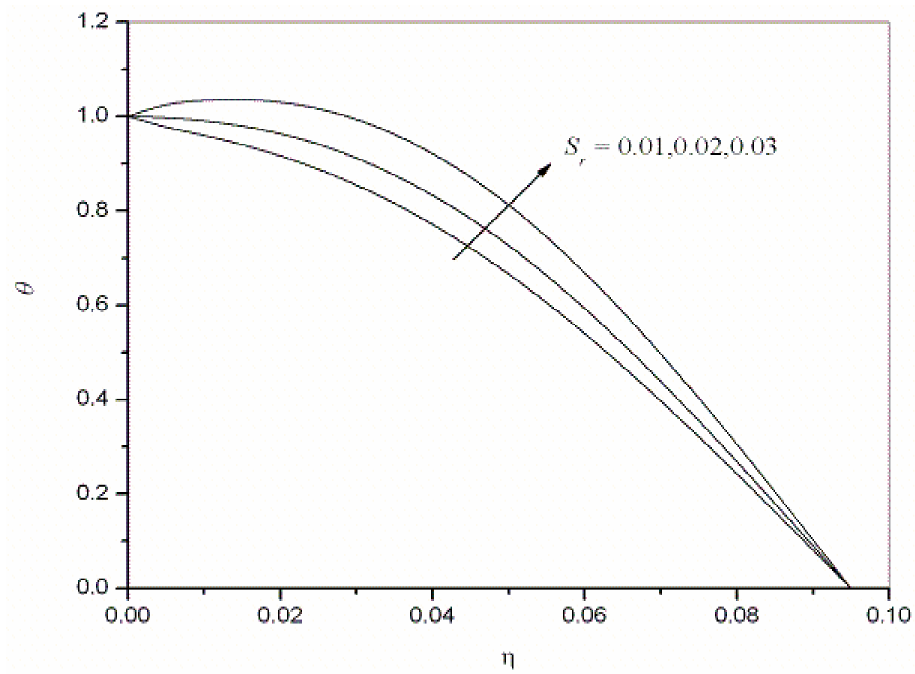

Fig. 6. Temperature profiles for different Soret parameters with $D_f = 10.0, N = 1$ **.**

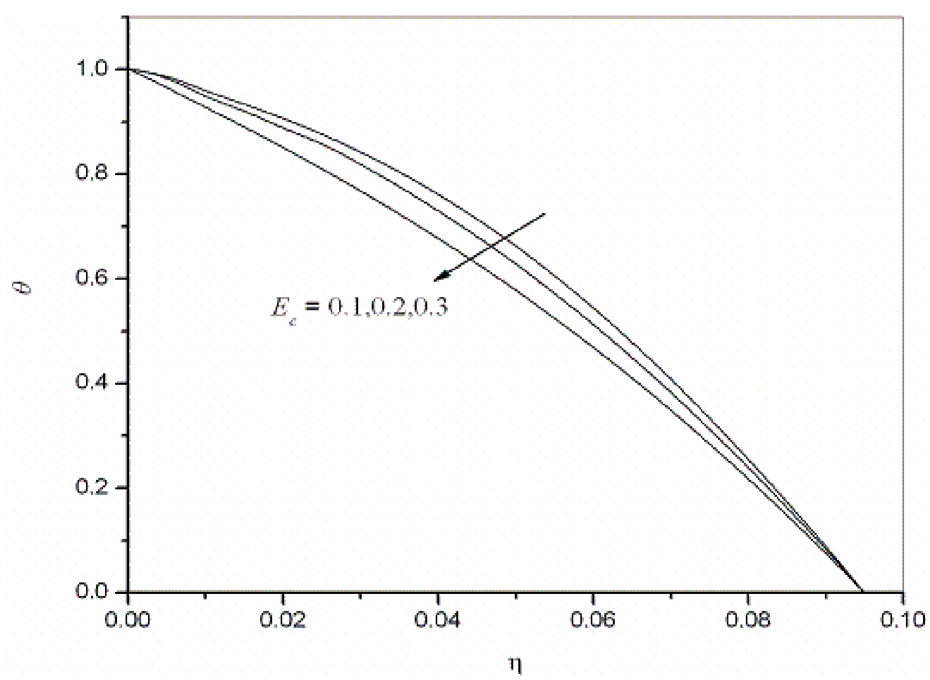

Fig. 7. Temperature profiles for different Eckert parameters E_c **with** $S_r = 0.001, D_f = 10.0$

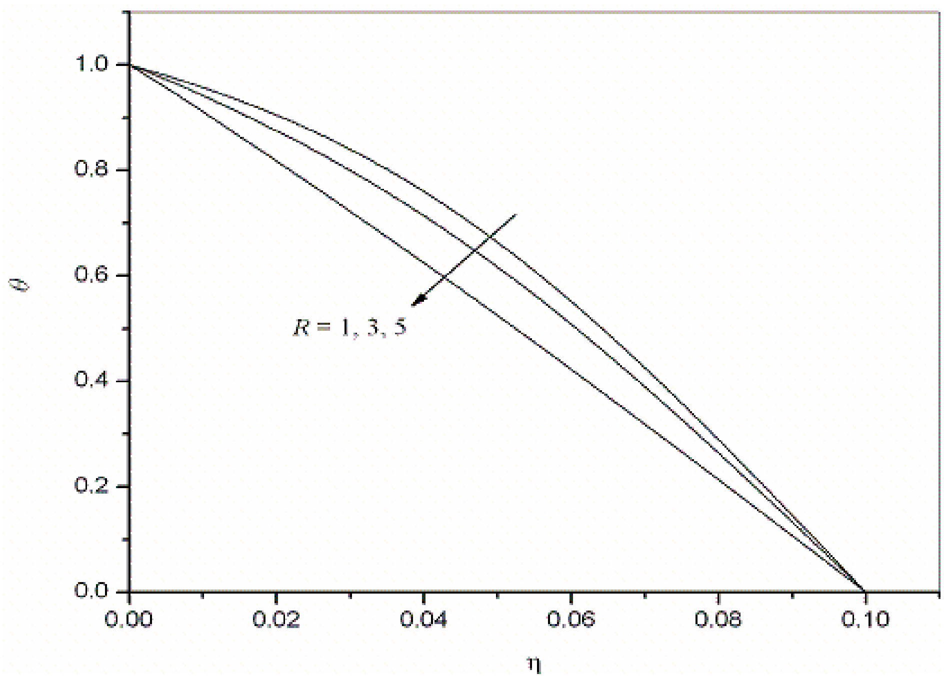

Fig. 8. Temperature profiles for different values of *R* with $S_r = 0.001, D_f = 10.0, N = 1, E_c = 0.5, Le = 2$

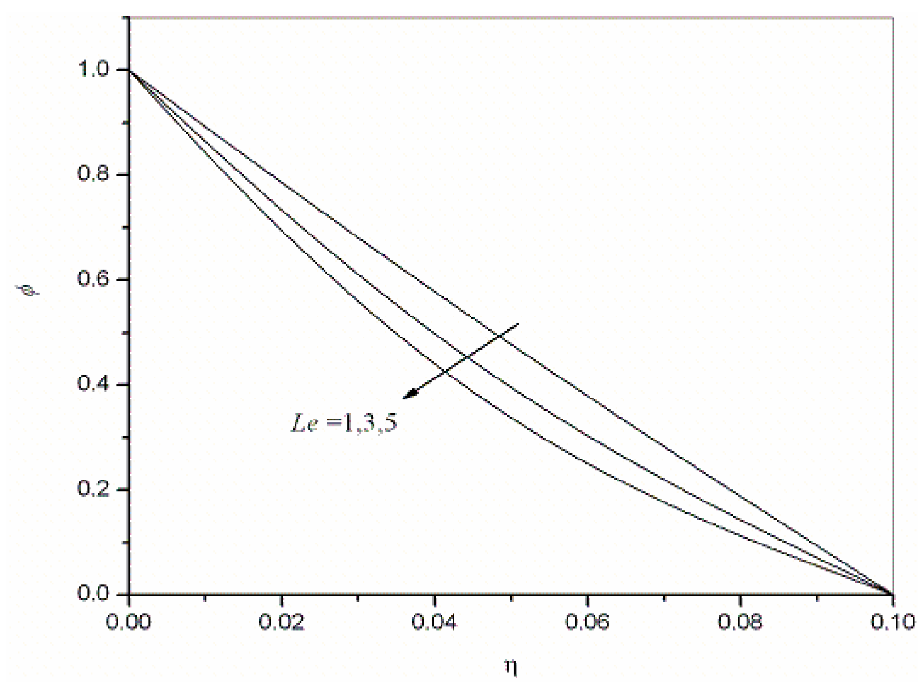

Fig. 9. Concentration profiles for different Le **with** $D_f = 10.0, S_r = 0.001$

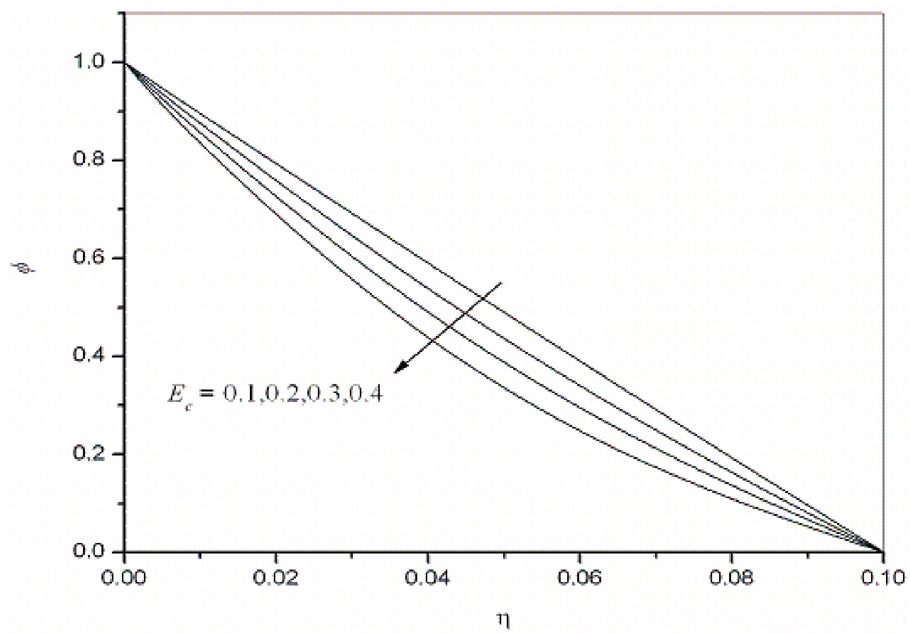

Fig. 10. Concentration profiles for different Ec **with** $D_f = 10.$ **and** $S_r = 0.001$

5. CONCLUSION

In the present study, we obtained numerical solution for two dimensional steady free convection flows of heat and mass transfer from vertical surface embedded in a darcy porous medium with viscous dissipation in the presence of thermal radiation. Following are the conclusions of the present study.

- The effect of radiation is to decrease the rate of energy transport to the fluid, there by decreasing the temperature of the fluid and the thermal radiation enhances convection flow.
- As the viscous dissipation increases, the velocity profiles increases where as concentration and temperature profiles decreases.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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