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Generalized Frequency Re-use Ratio for Uniform and Non-uniform Cell Range in Telecommunication Network Design

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Authors' contributions

This work was carried out in collaboration between all authors. Authors EKD and JAA designed the study and wrote the first and revised draft of the manuscript. Author SGA proof read the first and second draft of the manuscript. All authors read and approved the final manuscript.

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Abstract

GSM network design requires efficient interference management technique, which offers significant capacity enhancement and improves cell edge coverage with low complexity of implementation. This is done by assigning different frequencies to adjacent cells to avoid interference or cross talk. Random assignment of these frequencies is quite herculean and inefficient for huge number of cells. This paper proposed a formula for assigning frequencies for uniform $(N\colon N = 1\colon 1)$ cell range and extends it to non-uniform $(N\colon \aleph \text{ for } \aleph > N)$cell range in cell planning. Also, we obtain a functional relationship between the apothem and the circumradius as well as the inner and outer angle and deduce that hexagonal tessellation offers the best radius and angular relationship in GSM cell planning.

Keywords: Frequency re-use; GSM, hexagons; overlap difference.

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1 Introduction

In mobile telecommunication networks, the term 'cells' may artificially be represented by geometrical shapes such as hexagons or concentric circles-disks. This is because GSM antenna has been designed to radiate signals in sectors [1]. A collection of these antennas make up a circle. But circles do not tile, so the choice of covering is essential to remove excessive overlaps. Hexagon is conveniently chosen because it is the only tessellable regular polygon that is closest to being circular with the widest area [2]. The circular shapes are themselves inconvenient as they have overlapping coverage areas. In reality, the ideal coverage of the power transmitted by the base station antenna is non-geometric because of inconsistency in signal strengths. A practical network will have cells of non-geometric shapes, with some areas not having the required signal strength for various reasons. The performance of the cellular system greatly depends on the spatial configuration of Base Stations (BSs). FR is useful in assigning frequencies in hexagonal cells for densely and sparsely geographical distribution of subscribers of GSM network.

2 Related Works

The fundamental feature of wireless cellular network is its frequencies re-use capability to maximum channel capacity and coverage density [3]. This technique splits an area into smaller regions without overlap for particular region utilizes the full frequencies range to avoid interference. As large geographic regions are split into smaller units to minimize line-of-sight signal loss propagation, to aid a mass number of active resources in that area [4]. All of the cell sites are connected to switches, which in turn linked to the public switching network.

The demand for mobile services has been rising exponentially. However, the bandwidth and frequency spectrum to support these mobile services is critically limited. Due to limited and competitive scarce resources, GSM service providers need new tools to efficiently and effectively optimize the networks. Many techniques can be employed including frequency re-use, cell splitting, dynamic resource allocation, adaptive cell size algorithm [5] on a geometrically optimized covering cells.

GSM network coverage is identical to Geometric Disks Covering (GDC) which is one of the most typical and well-studied problems in computational geometry [6] and geometric topology. [7] investigated several optimal patterns for unbounded areas with special constraints, e.g., connectivity among nodes, are proposed in the area of wireless networking.

In the work of [8] , the author developed a bound on the largest area of a hexagon H that can be covered (with simple intersection) by n congruent convex domains K, i.e., $a(H) \leq nh(K)$, where $a(H)$ is the area of H and $h(K)$ is the maximum inscribed hexagon area in K.

In reference [9] applied geometric disks covering optimization to water carrot in crop management. In reference [10] the authors designed optimal patterns for connected coverage in wireless networks with directional antennae. The authors in [11] investigated how to cover a bounded square with a small number of circles, i.e., 6-8 and [12] extended their work up to 30 circles. Their patterns are highly specific, i.e., a unit square can be optimally covered by n discs with a specific radius. Tiling according to authors in [13] with squares and equilateral triangles are very useful tools to study several structural and thermodynamical properties of a wide variety of solids. [14] investigated the number of nodes needed to cover a bounded area.

There are several classical papers on the problem of how large an area n congruent shapes can cover. However, neither squares nor rectangles yield optimal overlap for cell planning.The contribution of this paper is summarized as: The proposed technique is capable of achieving a higher system throughput gain compared with conventional forwarding relay system. This will maximize the cell splitting frequencies to boost network coverage probability. Nonetheless, it tends to investigate the single optimality in re-using the frequency from the well-known uniform to a non-uniform cell range using tillable hexagons in both overlap difference and overlap areas. Also, a relationship of overlap difference for two non-uniform cell range has been computed.

3 Computational Experience

In a cellular communication system, cell shape varies depending on geographic, environmental and network parameters such as terrain and artificial structures properties, base station location and transmission power, accessing techniques, etc. We shall study the possible geometry of the cell shapes as shown in Fig. 1.

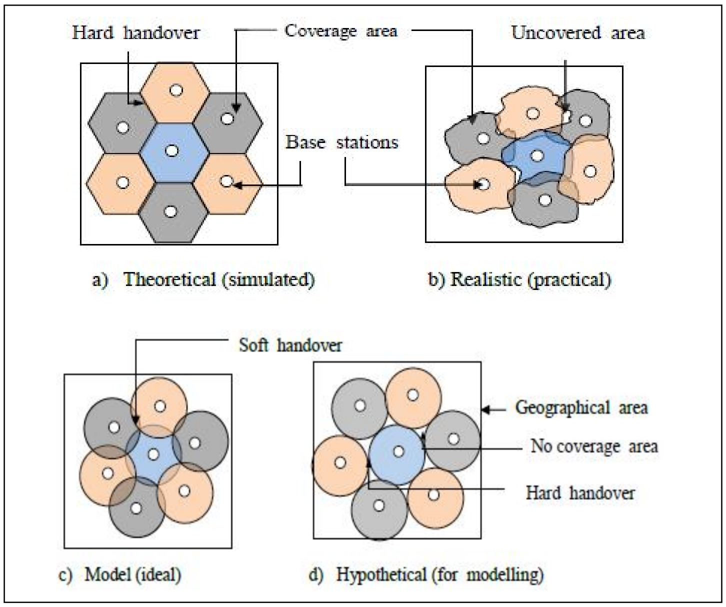


Fig. 1. GSM cell shapes in radio networks

3.1 Overlap for optimal disks covering

We consider simple layouts as shown in Fig. 2. Fig. 2(a) illustrates the hexagonal cell layout. The apothem and the circumradius of the hexagonal cell are r_1 and R_1, respectively. In Fig. 2(b), cells are partially overlapped because R_1 equals to the hexagon's circumradius. In this case, the model considers nodes were not belonging to the cell of interest. Cited in [15], algebraically, the best positioning of the GSM network is where the hexagonal and circular cells overlap to give us a difference of $2(R_1 - r_1)$ as shown in Fig. 2(a).

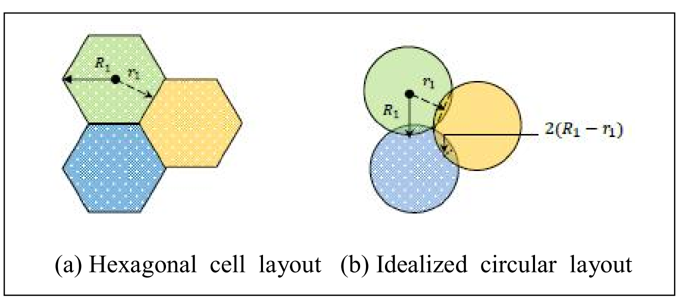


Fig. 2. Cell layout models for GSM networks

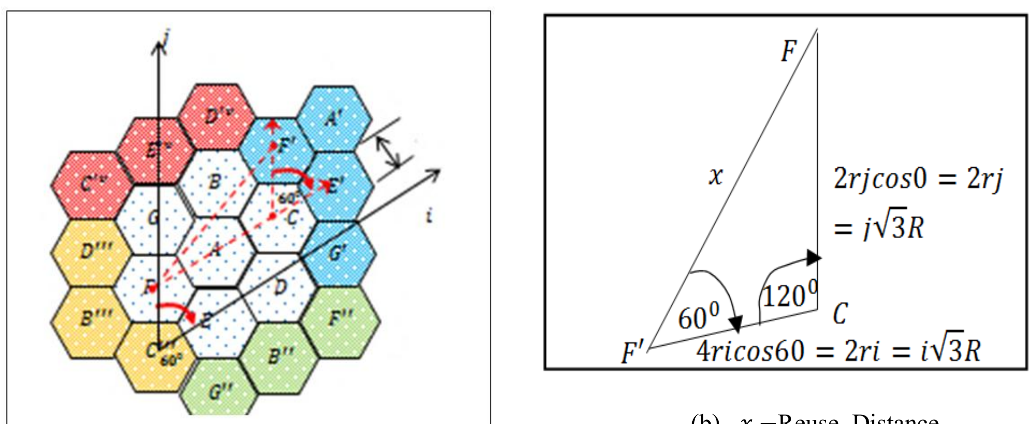
Theorem 1: For a hexagonal geometry of side ratio 1:1, the co-channel re-use ratio is given by

$$f_{ij} = \begin{cases} 3i & , \quad for \;\; i = j \\ \sqrt{3(i^2 + ij + j^2)}, & \quad for \;\; i \neq j \end{cases}$$

Where $N = i^2 + ij + j^2$ [10] is the cluster size for $i, j \in \mathbb{Z} \geq 0$.

Proof:

Consider the hexagonal geometric tessellation with seven different cells as shown in Fig. 3. The chain of hexagons is either along j vertical or 60^0 rotation of i cells.



(a) N- cluster cell ($N = 7$)

(b) x −Reuse Distance

Fig. 3. Co-channel reuse ratio

Generally, for $N = i^2 + ij + j^2$ we can find the nearest co-channel neighbours of a particular cell:

a) Move $i = 2$ cells along any chain of hexagons and then
b) Turn 120^0 clockwise and move $j = 1$ cells.
c) $N \in \mathbb{N}$ - Cluster size of cell
d) $R \in \mathbb{R}$ - radius of circle equivalent to any one side of the hexagon

Using the cosine rule to find x from ΔACF:

$$x^2 = (i\sqrt{3}R)^2 + (j\sqrt{3}R)^2 - 2i \times (\sqrt{3}R)j \times (\sqrt{3}R)cos120^0$$

$$= 3R^2i^2 + 3R^2j^2 - 6ijR^2\left(\frac{-1}{2}\right)$$

$$= 3R^2(i^2 + j^2 + ij)$$

$$x = R\sqrt{3(i^2 + j^2 + ij)} = |FF'|$$

For $i \neq j$ $\quad \therefore f_{ij} = \frac{x}{R} = \sqrt{3(i^2 + j^2 + ij)}$ (1)

For $i = j$, $\quad f_{i,i} = \frac{x}{R} = \sqrt{3(i^2 + i^2 + i \times i)} = 3i$

Is the reuse factor (ratio) and $x = R\sqrt{3(i^2 + j^2 + ij)}$ is the reuse distance. For a fixed cell size, if N is small then the cluster size decreases which in turn results in an increase in the number of clusters and hence the capacity. However, if N is small, the co-channel cells are located much closer and hence more interference.

Lemma 1: For a hexagonal geometry with $f_{ij} = \frac{x}{R} = \sqrt{3N}$ a small value of f_{ij} provides larger capacity since the cluster size is small (requires more of the same frequency) whereas because of a smaller level of co-channel interference a large value for f_{ij} improves the transmission quality. Table 1 shows some cluster size and frequency re-use factor of theorem 1.

Table 1. Possible cluster size and frequency re-use factor

Movement of cells	Cluster size (N)	Co-channel re-use ratio ($f_{ij} = \frac{x}{R} = \sqrt{3N}$)
$i = 1, j = 0$	1	1.732
$i = 1, j = 1$	3	3
$i = 2, j = 0$	4	3.46
$i = 2, j = 1$	7	4.58
$i = 3, j = 0$	9	5.20
$i = 2, j = 2$	12	6
$i = 3, j = 1$	13	6.24

For broadband cellular access based on orthogonal frequency division multiple access, fractional frequency reuse (FFR) is crucial in justifying inter-cell interference and optimizing cell-edge performance. The studies done in [16] about FFR, each cell is split into a centre zone, and an edge zone with their spectrum correspondingly partitioned into two parts. One part is allocated with re-use 1 in all cell-centre zones. The second part is further split into sub-bands. These sub-bands, to be used in the cell-edges zones, have a higher reuse factor. We, however, state a theorem is permitting us to re-use co-channel cells of hexagons of different side lengths.

Theorem 2: For a hexagonal geometry with side ratio $N: \aleph$, the co-channel re-use ratio for non-uniform cell range is given by

$$f_{ij} = \sqrt{3(N^2 i^2 + N\aleph ij + \aleph^2 j^2)} \quad \text{, for } \aleph \geq N$$

Where $n = N^2 i^2 + N\aleph ij + \aleph^2 j^2$ is the cluster size for $i, j \in 0 \cup \{\mathbb{N}\}$ and $N > \aleph \in N$.

Proof

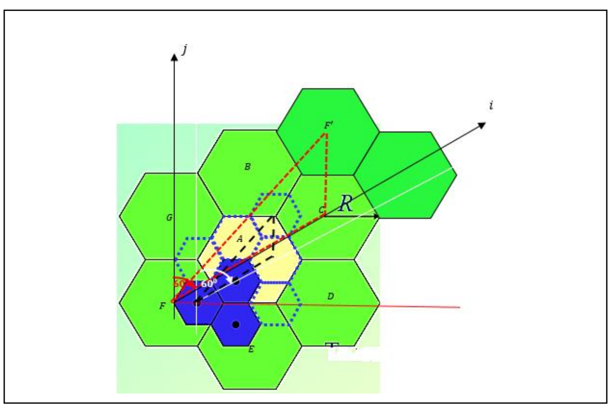


Fig. 4. Generalized frequency re-use in GSM network

Movement along the ith cells $= 4Nricos60^0$

$$= 2Nri = 2Ni\frac{\sqrt{3}}{2}R$$
$$= \sqrt{3}NRi$$

Movement along the jth cells $= 2\aleph rjcos0^0$

$$= 2\aleph rj = 2\aleph\,\frac{\sqrt{3}}{2}Rj$$
$$= \sqrt{3}\aleph Rj$$

Then,

$$x^2 = \left(iN\sqrt{3}R\right)^2 + \left(j\aleph\sqrt{3}R\right)^2 - 2\left(N\sqrt{3}R\right)i\left(\aleph\sqrt{3}R\right)jcos120^0$$
$$x = R\sqrt{3(N^2i^2 + N\aleph ij + \aleph^2j^2)}$$
$$f_{ij} = \frac{x}{R} = \sqrt{3(N^2i^2 + N\aleph ij + \aleph^2j^2)}$$

where $n = N^2i^2 + N\aleph ij + \aleph^2j^2$

Case 1: Uniform cell range: $N{:}\aleph = 1{:}1$

$n = i^2 + ij + j^2$ as in Theorem 1

Case 2: Non-uniform cell range: $N{:}\aleph = 1{:}2$

Then, $N = 1$ and $\aleph = 2$

$$n = i^2 + 2ij + 4j^2$$

Case 3: Non-uniform cell range: $N{:}\aleph = 1{:}3$

Then, $N = 1$ and $\aleph = 3$

$$n = i^2 + 3ij + 9j^2$$

Table 2. Generalized cluster size and frequency re-use factor

Movement of cells (i,j)	**$N{:}\aleph = 1{:}2$**		**$N{:}\aleph = 1{:}3$**	
	Cluster size (n)	**Co-channel re-use Ratio $(f_{ij} = \sqrt{3n})$**	**Cluster size (n)**	**Co-channel re-use Ratio $(f_{ij} = \sqrt{3n})$**
(1,0)	1	1.732	1	1.732
(0,1)	4	3.464	9	5.196
(1,1)	7	4.583	13	6.245
(2, 0)	4	3.464	4	3.464
(0, 2)	16	6.928	36	10.392
(2, 1)	12	6	19	7.550
(1, 2)	21	7.937	43	11.358
(3, 0)	9	5.196	9	5.196
(0, 3)	36	10.392	81	15.588
Sequence	1, 4, 7, 9, 12, 16, 21, ..		1,4,9,13,19,36,43,81, ...	

3.2 Overlap difference in hexagon-inscribed disks

Overlap in cell planning ensures smooth handover in GSM network. This overlap has no differential effect such as fading and attenuation in signals. The overlap may occur for a smooth handover of cells and has the disadvantage of increasing the number of GSM masts as well as antenna required in a given area. A typical overlap may arise as a result of uniform cell radius (disks) or non-uniform cell radius.

Type I: Uniform Disks

It has been established by [17], that to cover a given plane with disks of radius R_1 and hexagonal apothem r_1, we require an overlap difference of $2(R_1 - r_1)$. We deduce formulas for calculating the width of any hexagonal disks covering as shown in Table 2. Consider two intersecting uniform disks shown in Fig. 5.

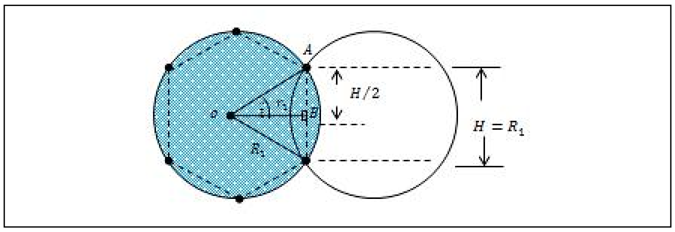


Fig. 5. Overlap width for uniform disks (cell radius)

Type II: Non-uniform Disks

Non-uniform cell radius for two different GSM antenna masts with radii R and R_1 and corresponding apothem of r and r_1 would have a cell overlap difference of $R - r + 2R_1 - r_1$. A mixture of non-uniform cell radius results in many overlaps and in effect give rise to an increase in the number of GSM antenna mast to be erected. Here we have relatively wider area. The difference $R - r + 2R_1 - r_1 > 2(R_1 - r_1)$. This is illustrated in Fig. 6.

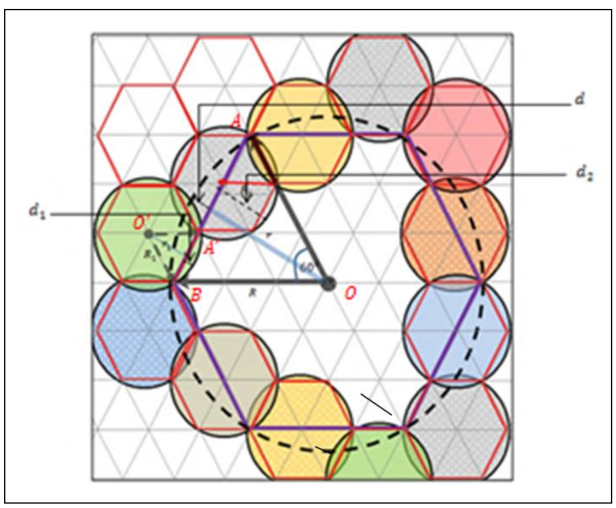


Fig. 6. Overlap difference for non-uniform disks

The actual expression for the overlap difference can be calculated from Fig. 6.

Overlap difference $= R - r + R_1 - r_1 + R_1$.

Case I: Triangle AOB

Overlap difference $(d) = R - r$

$$Sin60^0 = \frac{r}{R} \Rightarrow r = Rsin60^0$$

$$\therefore r = \frac{R\sqrt{3}}{2}$$

Overlap difference $(d) = \left(1 - \frac{\sqrt{3}}{2}\right)R = \frac{1}{2}\left(2 - \sqrt{3}\right)R$

Case II: Triangle $A'O'B$

Overlap difference $(d_1) = R_1 - r_1$

Similarly, triangle $A'O'B$ is equilateral so $r_1 = \frac{R_1\sqrt{3}}{2}$. Thus $d_1 = \frac{1}{2}\left(2 - \sqrt{3}\right)R_1$.

Case III: Circle with diameter AA'.

Overlap difference of $d_2 = R_1$. Generally, for non-uniform cell range, we have overlap difference

$$\begin{aligned} d_n &= d + d_1 + d_2 \\ &= R - r + R_1 - r_1 + R_1. \\ &= \tfrac{1}{2}\left(2 - \sqrt{3}\right)R + \tfrac{1}{2}\left(2 - \sqrt{3}\right)R_1 + \tfrac{2R_1}{2} \end{aligned}$$

$$d_n = \frac{1}{2}\left[\left(2 - \sqrt{3}\right)R + \left(4 - \sqrt{3}\right)R_1\right] \qquad (2)$$

Equation (2) is far greater than $2(R_1 - r_1)$. Thus it is inefficient to consider covering with disks using non-uniform radii. Applicably, masting of GSM antenna for non-uniform cell range is economically unwise as well as inefficient in time complexity. Table 3 illustrates the occupying overlap difference for both uniform and non-uniform cell radius.

Table 3. Occupying width for uniform and non-uniform cell range

GSM Cell Design Type	Uniform Cell Radius	Non-uniform Cell Radius
Overlap Difference (d_n)	$2(R_1 - r_1)$	$R - r + 2R_1 - r_1$
	$\left(2 - \sqrt{3}\right)R_1$	$\frac{1}{2}\left[\left(2 - \sqrt{3}\right)R + \left(4 - \sqrt{3}\right)R_1\right]$
	$\frac{2}{3}\left(2\sqrt{3} - 3\right)r_1$	$\frac{1}{3}\left[\left(2\sqrt{3} - 3\right)r + (4\sqrt{3} - 3)r_1\right]$

It is also an established fact that $R_1 > r_1$ and as R_1 increases the overlap difference (width - d_n) increases. This is because the multipliers $(2 - \sqrt{3})$ and $\left(4\sqrt{3} - 3\right)$ for both uniform and non-uniform disks respectively are both greater than one (1), hence as $R_1 = f(r_1) \to \infty$, then $d_n \to \infty$. The resulting area is calculated by the following approaches.

3.3 Area of single overlap

Overlap areas in hexagonal tessellation can be created using either uniform or non-uniform cell range. We consider two cases.

Case I: Uniform Cell radius

We have established that optimal disks covering is achieved when the cells overlap to give us a difference of $2(R_1 - r_1)$.

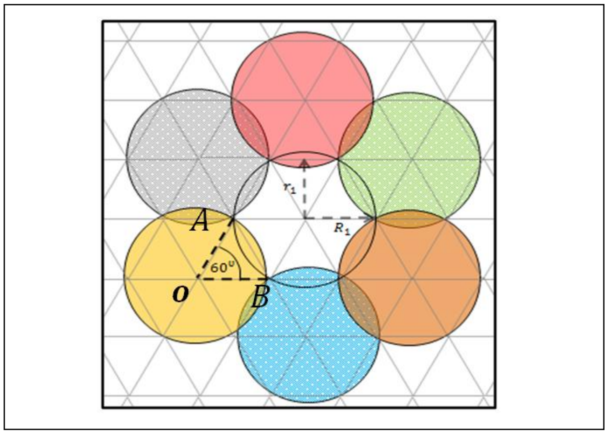


Fig. 7. Area of a single overlap for uniform disks

From Fig. 7 we have:

Area of single overlap = $2 \times$ (area of sector $AOB -$ an area of triangle AOB).

$$= 2 \times \left(\frac{1}{2}{R_1}^2\theta - \frac{1}{2}{R_1}^2 sin\theta\right)$$

$$A_s = {R_1}^2(\theta - sin\theta) \tag{3}$$

For an n −sided regular polygon, the total overlap area (A_T) is given by

$$A_T = n{R_1}^2(\theta - sin\theta) \tag{4}$$

For hexagonal tiling, $\theta = \frac{\pi}{3}$ a single area is

$$A_s = {R_1}^2\left(\frac{\pi}{3} - sin\frac{\pi}{3}\right)$$

$$A_s = \frac{R_1^2}{6}(2\pi - 3\sqrt{3}) \tag{5}$$

The achieved (5) is the formula for calculating excess area coverage loss (due to overlaps) when a pair of GSM masts are positioned with an overlap difference of $2(R_1 - r_1)$. The multiplier $0 < \frac{(2\pi - 3\sqrt{3})}{6} < 1$ widens the quadratic relationship between the area overlap and the radius of the inscribed hexagon in (5). In field work, an overlap has the differential disadvantage of reducing the coverage area to be covered. As a result the overlap area must be kept as small and few as possible - optimization. (5) can be modified using the fact that $R_1 = \frac{2r_1}{\sqrt{3}}$, then

Area of single overlap $= \frac{4r_1^2}{3} \times \frac{1}{6}(2\pi - 3\sqrt{3})$

$$= \frac{2r_1^2}{9}(2\pi - 3\sqrt{3}) \tag{6}$$

Case II: Non-uniform Cell Radius

Non-uniform cell radius for two different GSM antenna masts with radii R and R_1 with corresponding apothem of r and r_1 would have a cell overlap difference of $R - r + 2R_1 - r_1$. This is illustrated in Fig. 8.

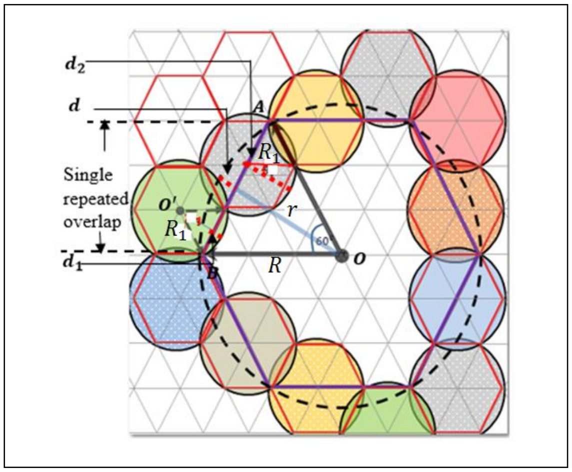


Fig. 8. Area of overlap for non-uniform disks

Area of a single repeated overlap

$A_{s'} =$ area of bigger sector $AOB -$ area of $\Delta AOB +$ area of smaller sector

$A'O'B -$ area of $\Delta A'O'B +$ (area of circle with diameter $AA' \div 2$)

$$= \frac{1}{2}R^2\theta - \frac{1}{2}R^2 sin\theta + \frac{1}{2}{R_1}^2\theta_1 - \frac{1}{2}{R_1}^2 sin\theta_1 + \frac{1}{2}\pi R_1^2$$

$$A_{s'} = \frac{1}{2}\left[R^2(\theta - sin\theta) + {R_1}^2(\theta_1 - sin\theta_1) + \frac{\pi}{2}R_1^2\right]$$

$$= \frac{1}{2}R^2\left[\frac{\pi}{3} - sin\left(\frac{\pi}{3}\right)\right] + \frac{1}{2}{R_1}^2\left(\frac{\pi}{3} - sin\frac{\pi}{3}\right) + \frac{\pi}{2}R_1^2$$

$$A_{s'} = \frac{1}{12}\left[R^2\left(2\pi - 3\sqrt{3}\right) + {R_1}^2\left(5\pi - 3\sqrt{3}\right)\right] \quad (7)$$

Equation (7) is repeated six (6) times for non-uniform disks covering. Hence the required total area is

$$A_{s'} = \frac{6}{12}\left[R^2\left(2\pi - 3\sqrt{3}\right) + {R_1}^2\left(5\pi - 3\sqrt{3}\right)\right] \quad (8)$$

(8) can be defined in terms of (5) connecting the two different areas. The relationship is

$$A_{s'} = \frac{1}{2}\left[R^2\left(2\pi - 3\sqrt{3}\right) + 3\pi{R_1}^2 + {R_1}^2\left(2\pi - 3\sqrt{3}\right)\right]$$

$$A_{s'} = \frac{1}{2}\left[R^2\left(2\pi - 3\sqrt{3}\right) + 3\pi{R_1}^2 + A_s\right] \quad (9)$$

Since $R > 0$ and $R_1 > 0$ the expression $R^2(2\pi - 3\sqrt{3}) + 3\pi {R_1}^2 >> 0$. However, $R^2(2\pi - 3\sqrt{3}) + 3\pi {R_1}^2 > A_s$, implying that $A_{s'} = A_s$; therefore making the value of $A_{s'} > A_s$. As a result the area of a single overlap difference in the uniform cell range as in (5) is smaller than that of the non-uniform cell range as in (9). It is not cost efficient for telecom engineers as well as industrial mathematicians to consider non-uniform GSM cell radius with number of intersecting cells more than that with an overlap difference of $2(R_1 - r_1)$. We find a relationship between the inner angle θ and the outer angle θ_1 as well as the inner radius R and outer radius R_1 respectively for a regular polygon of side n.

Case I: $\theta_1 = k_n\theta$

Where

$$k_n \begin{cases} > 1, for\; n > 6 \\ = 1, for\; n = 6 \\ < 1, for\; n < 6 \end{cases}$$

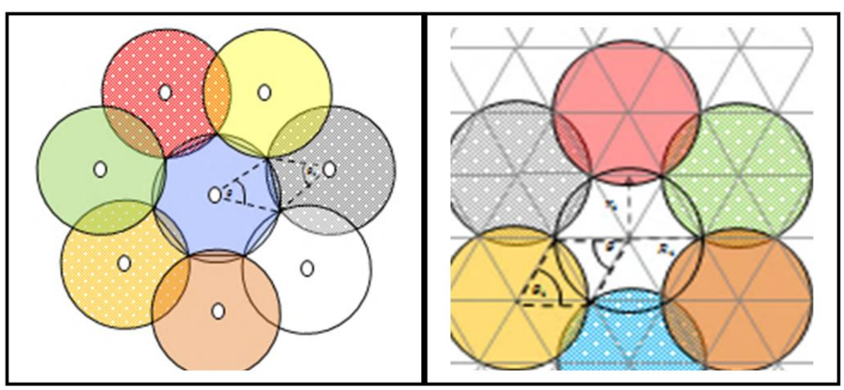
Geometrically,

Case II: $R = f(R_1)$ i.e. $R_1 : f^{-1} \rightarrow R$

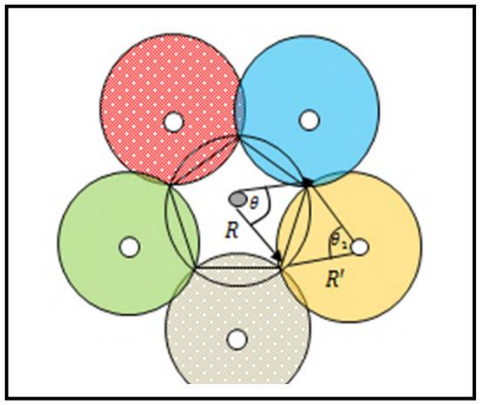
There is a linear relationship between R and R_1. For a GSM with large cell radius as centre and small cell radius as rings covering it circumference as in Fig. 9 we have the mathematical linear relationship

$$R = k_n R_1 \tag{10}$$

where $K_n \begin{cases} > 1, for\; n > 6 \\ = 1, for\; n = 6. \\ < 1, for\; n < 6 \end{cases}$



(a) Pentagon$(\theta_1 < \theta)$ (b) Hexagon$\left(\theta_1 = \frac{\pi}{3}\right)$



(c) Heptagon ($\theta_1 > \theta$)

Fig. 9. Angular and Radii relationship for uniform and non-uniform disks

Table 4. Illustrates example of the linear relationship between R_1 and R in equilateral triangle, square, pentagon and hexagon

Polygon	Equilateral triangle	Square	Pentagon	Hexagon
Sides	3	4	5	6
Angle at centre (θ, θ_1)	$\frac{2\pi}{3}$, $\theta_1 < \frac{2\pi}{3}$	$\frac{\pi}{2}, \theta_1 < \frac{\pi}{2}$	$\frac{2\pi}{5}$, $\theta_1 < \frac{2\pi}{5}$	$\frac{\pi}{3}$, $\theta_1 = \frac{\pi}{3}$
Overlap area	$\frac{1}{2}\begin{bmatrix}1.23R^2 + R_1^2 \\ (\theta_1 - sin\theta_1)\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}0.57R^2 + R_1^2 \\ (\theta_1 - sin\theta_1)\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}0.31R^2 + R_1^2 \\ (\theta_1 - sin\theta_1)\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}0.18R^2 + R_1^2 \\ (\theta_1 - sin\theta_1)\end{bmatrix}$
Radii - R, R_1	$R, R\sqrt{3}$,	$R, R\sqrt{2}$	$R, \frac{R}{2}\sqrt{10 - 2\sqrt{5}}$	R, R
Overlap area	$\frac{1}{2}\begin{bmatrix}1.23R^2 + 3R^2 \\ (\theta_1 - sin\theta_1)\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}0.57R^2 + 2R^2 \\ (\theta_1 - sin\theta_1)\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}0.31R^2 + 1.38 \\ R^2\ (\theta_1 - sin\theta_1)\end{bmatrix}$	$0181R^2$

4 Discussion of Results

In this paper we discussed the ratio of area of tessellable polygons to that of a circle. We established the fact that among the three tessellable regular polygons the area of hexagon approximates that of a circle more closely than any other regular tessellable polygon. This proof was further confirmed with the least overlap area $0.181R^2$ as shown in Table 4. The study lead us to a formula for calculating the generalized co-channel re-use ratio. Variant overlap difference was obtained for both uniform and non-uniform cell range and it was found that $2(R_1 - r_1) < R - r + 2R_1 - r_1$. The overlap area for uniform cell range is known to be calculated using $\frac{R_1^2}{6}(2\pi - 3\sqrt{3})$ or $\frac{2r_1^2}{9}(2\pi - 3\sqrt{3})$ and that of the non-uniform cell range is $\frac{1}{12}\left[R^2(2\pi - 3\sqrt{3}) + {R_1}^2(5\pi - 3\sqrt{3})\right]$. The study also establishes that a cell site planed with $n < 6$ will have both apothem and radius to be related of the form $R < R_1$ and inner and outer angle at centre of the form $\theta_1 < \theta$. Nonetheless, when $n > 6$, $R > R_1$ and $\theta_1 > \theta$ otherwise $R = R_1$ and $\theta_1 = \theta$. Table 4 establishes this relationships for the radius, angle and area of overlaps for sample polygons.

5 Conclusion

We found that GSM cell design requires a lot of geometric and algebraic calculations that aid in planning and analyzing of wireless networks. This was due to the fact that overlap difference for uniform cell range in GSM design network $2(R_1 - r_1)$ is far less than that of the non-uniform cell range $R - r + 2R_1 - r_1$. This result was consistent to the area coverage as uniform cell design has wider coverage area with fewer masts than non-uniform cell design. We use geometry to establish a formula for the co-channel re-use ratio and the cluster size which is systematic and transparent as compared to the arbitrary selection of frequency in telecommunication network design. An industrial mathematician or a cell planner needs to have an in-depth mathematical skill when deciding how to assign frequencies to cells for efficient network coverage.

Competing Interests

Authors have declared that no competing interests exist.

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