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Dynamics Analysis of a Damped Non Uniform Beam Subjected to Loads Moving with Variable Velocity

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

Aims/ objectives : To obtain the analytical solutions of the governing fourth order partial differential equations with variable and singular coefficients of non-uniform elastic beams under constant and harmonic variable loads travelling at varying velocity.
Study design: The study makes use of the governing equation of beam incorporating some parameters.
Place and Duration of Study: Department of Mathematical Sciences, Adekunle Ajasin University, P.M.B 01, Akungba-Akoko, Nigeria, Federal University of Technology, Akure, Nigeria, between July 2016 and July 2017.
Methodology: The governing equation of the problem is a fourth order partial differential equation. In order to solve this problem, elegant technique called Galerkin's Method is used to reduce the governing fourth order partial differential equations with variable and singular coefficients to a sequence of second order ordinary differential equations.

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Results: The results show that response amplitudes of the non uniform beam decrease as the value of the axial force N increases. Furthermore, for fixed value of axial force N, the displacements of the simply supported non uniform beam resting on elastic foundations decrease as the foundation modulus K increases. The results further show that, for fixed N and K, it is observed that higher values of the load longitudinal frequency produce more stabilizing effects on the elastic beam.

Conclusion: Higher values of axial force N and foundation moduli K reduce the risk factor of resonance in a vibrating system. Also higher load longitudinal frequency produce more stabilizing effects on the elastic beam thereby reduce resonance in a vibrating system.

Keywords: Galerkin's method; non-uniform beam; concentrated loads; axial force; the (P-A-L) variable velocity; longitudinal frequency; resonance.

1 INTRODUCTION

For some decades, the study of dynamic response of structural members resting on elastic foundation subjected to moving loads is interesting and important, as some of the results may be applicable in understanding the dynamic behavior of roadways and runways. Among these is the work of Stanistic et al [1], Fryba [2], Sadiku and Leipholz [3], Adedowole [4], Oni [5], Oni and Adedowole [6] and Jimoh and Adedowole [7] to mention a few.

In the analysis of roadways and runways of airports above, the structure is usually modeled as beam or plate resting on an elastic foundation. In general, loads on these types of structures are loads moving with constant velocity such as the wheel loads from moving vehicles and planes. Hence, structural members on elastic foundation under moving loads with constant velocity have received considerable attention in the literature.

The more practical cases when velocities at which these loads move are no longer constants but vary with the time have received little attention in literature. This may be as a result of the complex space-time dependencies inherent in such problem. Specifically, even when such structures are non-uniform the analytical solutions involving integral transforms are both intractable and cumbersome. However, such practical problems as acceleration and braking of automobile on roadways and highway bridges, taking off and landing of air-crafts on runway and braking and acceleration forces in the calculation of rails and railway bridges in which the motion is not uniform but a function of time have intensified the need for the study of the behavior of structures under the action of loads moving with variable velocity. The class of problems was first tackled by Lowan [8] who solved the problem of the transverse oscillations of beams under the action of moving variable loads. Much later, Kokhmanyuk and Filippov [9] treated the dynamic effects on the transverse motion of a uniform beam of a load moving at variable speed. Wang [10] studied the dynamical analysis of a finite inextensible beam with an attached accelerating mass. He employed Galerkin procedure in conjunction with the method of numerical integration to tackle the partial differential equation which describes the transient vibration of the beam mass system. He concluded that the applied forward force amplifies the speed of the mass and the displacement of the beam.

In particular, Oni and Omolofe [11] worked on the dynamic Behavior of non-uniform Bernoulli-Euler Beams subjected to concentrated loads travelling at varying velocities. They obtained an analytical solution to the dynamical problem. For the illustrative classical boundary conditions considered, they found that for the same natural frequency, the critical speed for moving mass problem is smaller than that of the moving force problem. Hence, resonance is reached earlier in moving mass problem. The authors [12] also studied dynamic response of prestressed Rayleigh beam resting on elastic foundation and subjected to masses travelling at varying velocity. Omolofe and Ogunyebi [13] studied the dynamic behavior of a rotating Timoshenko beam when under the actions of a variable magnitude load moving at non-uniform speed. Adedowole [14] also consider flexural vibration of non prismatic Rayleigh beam with non uniform prestress under concentrated loads moving with variable velocity.

Thus, this work is concerned with the dynamics analysis of a damped non uniform beam under the actions of loads traveling with variable velocity. The main objective of this paper is to provide a closed form solution to this problem and to classify the effect of various parameters of the dynamical system on the response of the beam.

2 FORMULATION OF THE INITIAL BOUNDARY VALUE PROBLEM

The motion of a Bernoulli-Euler beam resting on an elastic foundation and under the action of a load moving with variable velocity is governed by the partial differential equation

$$-\frac{\partial Q(x,t)}{\partial x}+\mu(x)\frac{\partial^2 w(x,t)}{\partial t^2}-N(x)\frac{\partial^2 w(x,t)}{\partial x^2}+b(x)\frac{\partial w(x,t)}{\partial t}+Kw(x,t)-q(x,t)=0 \tag{2.1}$$

$$Q(x,t)=\frac{\partial D(x,t)}{\partial x} \tag{2.2}$$

Where $Q(x,t)$ is the shear force, $q(x,t)$ is the constant moving concentrated load moving with variable velocity acting on the beam, μ is the mass of the beam per unit length L,b is the material damping intensity, $w(x,t)$ is the vertical response of the beam, $D(x,t)$ is the flexural moment and t is time. The flexural moment acting on the beam across section is related to the vertical response as

$$D(x,t)=-EI(x)\frac{\partial^2 w(x,t)}{\partial x^2} \tag{2.3}$$

Where $EI(x)$ the flexural rigidity of the beam, E is the young modulus

$N(x)$ is the non-uniform axial force, x and t are the spatial and time coordinates respective. The structure under consideration is simply supported and carrying a concentrated mass M, which is moving at variable velocity.

2.1 The Boundary Conditions

The boundary conditions depend on the constraints at the beam ends, however for a simply supported beam whose length is L, the vertical displacement at the beam ends are given as:

$$w(0,t)=w(L,t)=0 \tag{2.4}$$

$$w''(0,t)=w''(L,t)=0 \tag{2.5}$$

where dash means derivative with respect to x

It is assumed that the initial conditions are

$$w(x,0)=0=\frac{\partial^2 w(x,0)}{\partial t^2} \tag{2.6}$$

The body moves with non-uniform velocity such that the motion of the contact of the moving load is given by

$$X_p=f(t) \tag{2.7}$$

The distance covered by the load on the same structure at any given instance of time t is given as

$$f(t)=x_o+x_1 \tag{2.8}$$

where x_0 and x_1 are initial position and position at any of the moving load. For the variable moment of inertia I, the mass per unit length μ , and the material damping intensity b of the beam, we adopt the example in [14] and take I(x), $\mu(x)$ and $b(x)$ to be of the form,

$$I(x) = I_o\left(1+\sin\frac{\pi x}{L}\right)^3 \tag{2.9}$$

$$\mu(x) = \mu_o\left(1+\sin\frac{\pi x}{L}\right) \tag{2.10}$$

and

$$b(x) = b_o\left(1+\sin\frac{\pi x}{L}\right) \tag{2.11}$$

In this paper, the non-uniform axial force is defined as [15] $N(x)$

$$N(x) = N_o\left(1+\sin\frac{\pi x}{L}\right) \tag{2.12}$$

In what follows, two special cases of equation (2.1) are considered. They are termed constant load and Harmonic load problems.

2.2 Case I

2.2.1 The dynamic response of non uniform beam under the actions of constant magnitude mobile concentrated forces

The concentrated mobile force $q(x,t)$ in equation (2.1) is assumed to be moving at variable velocities is given by

$$q(x,t) = P_o\delta\{x-(x_o+x_1)\} \tag{2.13}$$

and x_1 is defined as

$$x_1 = \beta\sin\alpha t \tag{2.14}$$

where x_o is the equilibrium position of the longitudinal oscillating load, $\beta$is the longitudinal amplitude of oscillation of the load and α is the longitudinal frequency of the load.

When equations (2.2),(2.3), (2.9), (2.11),(2.12) and (2.13) are substituted into equation (2.1), the result is a non- homogeneous system of partial differential equation with variable coefficients given by

$$\begin{aligned} EI_o\frac{\partial^2}{\partial x^2}\left[\left(1+\sin\frac{\pi x}{L}\right)^3\frac{\partial^2 w(x,t)}{\partial x^2}\right]+\mu_o\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^2 w(x,t)}{\partial t^2}+b_o\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial w(x,t)}{\partial t} \\ -N\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^2 w(x,t)}{\partial x^2}+K_0 w(x,t) = P_o\delta\{x-(x_o+\beta\sin\alpha t)\} \end{aligned} \tag{2.15}$$

In the equation (2.15) above, a closed from solution to the fourth order partial differential equation governing the motion of the beam under the action of concentrated moving does not exist. Consequently, an approximate analytical solution is desirable to obtain some vital information about the vibrating system.

2.3 Approximate Analytical Solution

To solve the beam problem above in equation (2.15), we shall use the versatile solution technique called Galerkin's method often used in solving diverse problems involving mechanical vibrations [11]. The equation of the motion of an element of the beam is generally symbolically written in the form.

$$\Gamma w(x,t) - q(x,t) = 0 \tag{2.16}$$

where Γ is the differential operator with variable coefficients, $w(x,t)$ is the beam displacement, q(x,t) is the load acting on the beam, x and t are spatial coordinates and time respectively. The solutions of the system of equation (2.15) is expressed as

$$w(x,t) = \sum_{i=1}^{n} y_i(t)Q_i(x) \tag{2.17}$$

where $y_i(t)$ are coordinates in modal space and$Q_i(x)$are the normal modes of free vibration written as

$$Q_i(x) = \sin\theta_i x + A_i \cos\theta_i x + B_i \sinh\theta_i x + C_i \cosh\theta_i x \tag{2.18}$$

where the constant, $A_i, B_i\ and\ C_i$, define the space and amplitude of the beam vibration. Their values depend on the boundary condition associated with the structure. Thus, for a simply supported beam, it can be shown that

$$A_i = B_i = C_i = 0 \qquad and \qquad \theta_i = \frac{i\pi}{L} \tag{2.19}$$

Thus, for a beam with simple supports at both ends, equation (2.18) takes the form

$$Q_i(x) = \sin\frac{i\pi x}{L} \tag{2.20}$$

Thus in view of equation (2.20) the transverse displacement response of a simply supported elastic beam, using an assumed mode method can be written as

$$w(x,t) = \sum_{i=1}^{n} y_i(t)\sin\frac{i\pi x}{L} \tag{2.21}$$

Substituting equation (2.21) into the governing equation (2.15) and after some simplifications and arrangements one obtains

$$\begin{aligned}
&\frac{EI_o}{4}\left[\left(10 - 6\cos\frac{\pi x}{L} + 15\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)\frac{\partial^4}{\partial x^4}\sum_{i=1}^{\infty} y_i(t)\sin\frac{i\pi x}{L}\right] \\
&+\frac{3\pi}{L}\left(\sin\frac{3\pi x}{L} + 5\cos\frac{\pi x}{L} - \cos\frac{3\pi x}{L}\right)\frac{\partial^3}{\partial x^3}\sum_{i=1}^{\infty} y_i(t)\sin\frac{i\pi x}{L} \\
&+3\left(\frac{\pi}{L}\right)^2\left(8\cos\frac{2\pi x}{L} - 5\sin\frac{\pi x}{L} + 3\sin\frac{3\pi x}{L}\right)\frac{\partial^2}{\partial x^2}\sum_{i=1}^{\infty} y_i(t)\sin\frac{i\pi x}{L} \\
&+\mu_o\left(1 + \sin\frac{\pi x}{L}\right)\frac{\partial^2}{\partial x^2}\sum_{i=1}^{n} y_i(t)\sin\frac{i\pi x}{L} + b_o\left(1 + \sin\frac{\pi x}{L}\right)\frac{\partial}{\partial t}\sum_{i=1}^{n} y_i(t)\sin\frac{i\pi x}{L} \\
&-N\left(1 + \sin\frac{\pi x}{L}\right)\frac{\partial^2}{\partial x^2}\sum_{i=1}^{n} y_i(t)\sin\frac{i\pi x}{L} + K\sum_{i=1}^{n} y_i(t)\sin\frac{i\pi x}{L} = P_o\delta\{x - (x_o + \beta\sin\alpha t)\}
\end{aligned} \tag{2.22}$$

Subjecting equation (2.22) to further simplification, one obtains

$$\begin{aligned}
&\frac{EI_o}{4}\left[R_1(x)\left(\frac{i\pi x}{L}\right)^4\sin\frac{i\pi x}{L} - R_2(x)\left(\frac{i\pi x}{L}\right)^3\cos\frac{i\pi x}{L} - R_3(x)\left(\frac{i\pi x}{L}\right)^2\sin\frac{i\pi x}{L}\right]y_i(t) \\
&+\mu_o R_4(x)\ddot{y}_i(t)\sin\frac{i\pi x}{L} + b_o R_4(x)\dot{y}_i(t)\sin\frac{i\pi x}{L} \\
&+N_o\left(\frac{i\pi}{L}\right)^2 R_4(x) y_i(t)\sin\frac{i\pi x}{L} + K y_i(t)\sin\frac{i\pi x}{L} = P_o\delta\{x - (x_o + \beta\sin\alpha t)\}
\end{aligned} \tag{2.23}$$

where

$R_1(x) = \left(10 - 6\cos\frac{\pi x}{L} + 15\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)$
$R_2(x) = \frac{3\pi}{L}\left(\sin\frac{3\pi x}{L} + 5\cos\frac{\pi x}{L} - \cos\frac{3\pi x}{L}\right)$
$R_3(x) = 3\left(\frac{\pi}{L}\right)^2\left(8\cos\frac{2\pi x}{L} - 5\sin\frac{\pi x}{L} + 3\sin\frac{3\pi x}{L}\right)$
$R_4(x) = \left(1 + \sin\frac{\pi x}{L}\right)$

To determine P$_i$ (t), the expression on the left hand side of equation (2.12) is required to be orthogona to the functions $\sin\frac{k\pi x}{L}$ Thus,

$$\int_0^L \sum_{i=1}^{n} \left\{ \frac{EI_o}{4} \left[R_1(x) \left(\frac{i\pi x}{L}\right)^4 \sin\frac{i\pi x}{L} - R_2(x) \left(\frac{i\pi x}{L}\right)^3 \cos\frac{i\pi x}{L} - R_3(x) \left(\frac{i\pi x}{L}\right)^2 \sin\frac{i\pi x}{L} \right] y_i(t) \right.$$

$$\left. +\mu_o R_4(x)\ddot{y}_i(t)\sin\frac{i\pi x}{L} + b_o R_4(x)\dot{y}_i(t)\sin\frac{i\pi x}{L} + N_o\left(\frac{i\pi}{L}\right)^2 R_4(x) y_i(t)\sin\frac{i\pi x}{L} + K y_i(t)\sin\frac{i\pi x}{L}\right\}\sin\frac{k\pi x}{L}$$

$$= \int_0^L P_o \delta\{x - (x_o + \beta\sin\alpha t)\}\sin\frac{k\pi x}{L} \tag{2.24}$$

From the right hand side of equation (2.24), we have
$\sin\alpha t = \alpha t - \frac{(\alpha t)^3}{3!} + \frac{(\alpha t)^5}{5!} - \frac{(\alpha t)^7}{7!} + \frac{(\alpha t)^9}{9!}$
Since $\alpha\, and\, t$ are considerably small, therefore $\sin\alpha t$ can be approximated to αt in this study

Equation (2.24) after some rearrangements and simplifications yields

$$D_1(i,k)\ddot{y}_i(t) + D_2(i,k)\dot{y}_i(t) + D_3(i,k)y_i(t) = P_o \sin\frac{k\pi(x_o + \beta\sin\alpha t)}{L} \tag{2.25}$$

where

$D_1(i,k) = \mu_o[I_1 + I_3]; \qquad D_2(i,k) = \varepsilon_o I_1; \qquad D_3(i,k) = \frac{EI_o}{4}[E_1 - E_2 - E_3] + E_4 + E_5$

$$E_1 = \left(\frac{i\pi}{L}\right)^4 [10I_1 - 6I_2 + 15I_3 - I_4]; \qquad E_2 = 3i^3\left(\frac{\pi}{L}\right)^4 [4I_5 + 5I_6 - I_7] \tag{2.26}$$

$$E_3 = 3i^2\left(\frac{\pi}{L}\right)^4 [8I_2 - 5I_3 + 3I_4]; \qquad E_4 = \left(\frac{i\pi}{L}\right)^2 [I_1 + I_3]; \qquad E_5 = KI_1 \tag{2.27}$$

The integrals I_i are as follow

$$I_1 = \int_0^L \sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx; \qquad I_2 = \int_0^L \cos\frac{2\pi x}{L}\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$$

$$I_3 = \int_0^L \sin\frac{\pi x}{L}\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx; \qquad I_4 = \int_0^L \sin\frac{3\pi x}{L}\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$$

$$I_5 = \int_0^L \sin\frac{2\pi x}{L}\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx; \qquad I_6 = \int_0^L \cos\frac{\pi x}{L}\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$$

$$I_7 = \int_0^L \cos\frac{3\pi x}{L}\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx \tag{2.28}$$

Equation (2.25) is a second order differential equation with constant coefficients.

In what follows, we subject the system of ordinary differential equation (2.25) to Laplace transform defined as

$$(\sim) = \int_\circ^\infty (\cdot)e^{-st}dt \tag{2.29}$$

where s is the Laplace parameter applying the initial condition (2.6), we obtain

$$(D_1(i,k)S^2 + D_2(i,k)S + D_3(i,k))y_i(s) = P_0\left[b_0\frac{S}{S^2+\gamma_o^2} - a_0\frac{\gamma_o}{S^2+\gamma_o^2}\right] \tag{2.30}$$

where

$$\gamma_o = \frac{k\pi\beta\alpha}{L}, a_o = \sin\frac{k\pi x_o}{L}, b_o = \cos\frac{k\pi x_o}{L} \tag{2.31}$$

Subjecting equation (2.30) to some simplifications and rearrangements gives

$$y_i(s) = P_0\left[b_0\frac{s}{s^2+\gamma_o^2} - a_0\frac{\gamma_o}{s^2+\gamma_o^2}\right]\frac{1}{(D_1(i,k)s^2 + D_2(i,k)s + D_3(i,k))} \tag{2.32}$$

which reduces to

$$y_i(S) = \frac{P_0}{(\beta_1-\beta_2)}\left(\frac{1}{S-\beta_1} - \frac{1}{S-\beta_2}\right)\left[b_0\frac{S}{S^2+\gamma_o^2} - a_0\frac{\gamma_o}{S^2+\gamma_o^2}\right] \tag{2.33}$$

where

$$\beta_1 = \frac{-D_2 + \sqrt{D_2^2 - 4D_1D_3}}{2D_1} \quad and \quad \beta_2 = \frac{-D_2 - \sqrt{D_2^2 - 4D_1D_3}}{2D_1} \tag{2.34}$$

To obtain the Laplace inversion of (2.33), the following representation is adopted.

$$g_1(s) = \frac{s}{s^2+\gamma^2}, \quad g_2(s) = \frac{\gamma}{s^2+\gamma^2}, \quad f_1(s) = \frac{\beta_1}{s-\beta_1} \quad and \quad f_2(s) = \frac{\beta_2}{s-\beta_2} \tag{2.35}$$

So that the Laplace invasion of (2.33) is the convolution of $f_i{}'$s and g defined by

$$f_s * g = \int_0^t f_i(t-u)g(u)du, \qquad i = 1,2 \tag{2.36}$$

Thus, the Laplace inversion of equation (2.33) is given by

$$y_i(t) = P_p\left[\frac{e^{\beta_1 t}}{\beta_1}(b_oI_8 - a_oI_9) - \frac{e^{\beta_2 t}}{\beta_2}(b_oI_{10} - a_oI_{11})\right] \tag{2.37}$$

where

$$P_p = \frac{P_0}{(\beta_1+\beta_2)} \tag{2.38}$$

$$I_8 = \int_0^t e^{-\beta_1 u}\cos\gamma u du; \qquad I_9 = \int_0^t e^{-\beta_1 u}\sin\gamma u du$$

$$I_{10} = \int_0^t e^{-\beta_2 u}\cos\gamma u du; \qquad I_{11} = \int_0^t e^{-\beta_2 u}\sin\gamma u du \tag{2.39}$$

Evaluating the integrals (2.39) above, we obtain

$$I_8 = \frac{1}{(\gamma^2+\beta_1^2)}\left(-\gamma e^{\beta_1 t}\sin\gamma t + \beta_1 - \beta_1 e^{\beta_1 t}\cos\gamma t\right) \tag{2.40}$$

$$I_9 = \frac{1}{(\gamma^2+\beta_1^2)}\left(-\gamma e^{-\beta_1 t}\cos\gamma t + \gamma - \beta_1 e^{-\beta_1 t}\sin\gamma t\right) \tag{2.41}$$

$$I_{10} = \frac{1}{(\gamma^2+\beta_2^2)}\left(-\gamma e^{\beta_2 t}\sin\gamma t + \beta_2 - \beta_2 e^{\beta_2 t}\cos\gamma t\right) \tag{2.42}$$

$$I_{11} = \frac{1}{(\gamma^2+\beta_2^2)}\left(-\gamma e^{-\beta_2 t}\cos\gamma t + \gamma - \beta_2 e^{-\beta_2 t}\sin\gamma t\right) \tag{2.43}$$

Subjecting equation (2.39) to some simplifications and rearrangements yields,

$$\begin{aligned} y_i(t) = P_p\Bigg\{\Bigg(&\frac{e^{\beta_1 t}}{\beta_1(\gamma^2+\beta_1^2)}a_o\left(-\gamma e^{\beta_1 t}\sin\gamma t + \beta_1 - \beta_1 e^{\beta_1 t}\cos\gamma t\right)\\ &\quad -b_o\left(-\gamma e^{-\beta_1 t}\cos\gamma t + \gamma - \beta_1 e^{-\beta_1 t}\sin\gamma t\right)\Bigg)\\ -\Bigg(&\frac{e^{\beta_2 t}}{\beta_2(\gamma^2+\beta_2^2)}a_o\left(-\gamma e^{\beta_2 t}\sin\gamma t + \beta_2 - \beta_2 e^{\beta_2 t}\cos\gamma t\right)\\ &\quad +b_o\left(-\gamma e^{-\beta_2 t}\cos\gamma t + \gamma - \beta_2 e^{-\beta_2 t}\sin\gamma t\right)\Bigg)\Bigg\} \end{aligned} \tag{2.44}$$

which on inversion yields

$$\begin{aligned} w(x,t) = \sum_{i=1}^{n} P_p\Bigg\{\Bigg(&\frac{e^{\beta_1 t}}{\beta_1(\gamma^2+\beta_1^2)}a_o\left(-\gamma e^{\beta_1 t}\sin\gamma t + \beta_1 - \beta_1 e^{\beta_1 t}\cos\gamma t\right)\\ &\quad -b_o\left(-\gamma e^{-\beta_1 t}\cos\gamma t + \gamma - \beta_1 e^{-\beta_1 t}\sin\gamma t\right)\Bigg)\\ -\Bigg(&\frac{e^{\beta_2 t}}{\beta_2(\gamma^2+\beta_2^2)}a_o\left(-\gamma e^{\beta_2 t}\sin\gamma t + \beta_2 - \beta_2 e^{\beta_2 t}\cos\gamma t\right)\\ &+b_o\left(-\gamma e^{-\beta_2 t}\cos\gamma t + \gamma - \beta_2 e^{-\beta_2 t}\sin\gamma t\right)\Bigg)\Bigg\}\sin\frac{i\pi x}{L} \end{aligned} \tag{2.45}$$

Equation (2.45) represents the transverse displacement response of the damped beam with non uniform axial force subjected to the action of fast constant mobile concentrated forces with variable velocities.

2.4 Case II

2.4.1 The dynamic response of damped beam under the actions of harmonic magnitude mobile concentrated forces with variable velocity

Here, the moving force q(x,t) is given as

$$q(x,t) = P_o \sin\omega t\,\delta\{x - (x_o + \beta\sin\alpha t)\} \tag{2.46}$$

where all parameters are as defined as before. Thus in view of equation (2.1) taking into account (2.46) one obtains

$$l-\frac{\partial Q(x,t)}{\partial x} + \mu(x)\frac{\partial^2 w(x,t)}{\partial t^2} + b(x)\frac{\partial w(x,t)}{\partial t} - N(x)\frac{\partial^2 w(x,t)}{\partial x^2} + Kw(x,t) = P_o\sin\omega t\,\delta\{x-(x_o+\beta\sin\alpha t)\} \tag{2.47}$$

Equation (2.47) is the governing equation describing the motion of non- uniform elastic beam subjected to mobile forces of varying magnitude. Like in the previous section, a closed form solution to equation (2.47) is sought. To this effect, use is made of an assumed mode method already alluded to and by this method, the transverse deflection $w_a(x,t)$ of non-uniform beam under the action of variable magnitude mobile force can be written as

$$w_a(x,t) = \sum_{m=1}^{\infty} y_m(t) Q_m(x) \tag{2.48}$$

where $y_m(t)$ are coordinates in modal space and $Q_m(x)$ are the normal modes of free vibration. Thus for a simply supported beam equation (2.48) becomes

$$w_a(x,t) = \sum_{m=1}^{\infty} y_m(t)\sin\frac{m\pi x}{L} \tag{2.49}$$

Using equation (2.49) in equation (2.47) and following the same arguments as in the previous section and after some simplifications and rearrangements one obtains

$$\sum_{m=1}^{\infty}\{D_1(m,k)\ddot{y}_m(t)+D_2(m,k)\dot{y}_m(t)+D_3(m,k)y_m(t)\} = P_o\sin\omega t\sin\frac{k\pi(x_o+\beta\sin\alpha t)}{L} \tag{2.50}$$

Without loss of generality, considering only the m^{th} particle of the dynamical system yields

$$D_1(m,k)\ddot{P}_m(t)+D_2(m,k)\dot{P}_m(t)+D_3(m,k)P_m(t) = P_o\sin\omega t\sin\frac{k\pi(x_o+\beta\sin\alpha t)}{L} \tag{2.51}$$

Subjecting equation (2.51) as defined previously yields

$$\begin{aligned} y_m(S) = P_p\Bigg\{&\frac{1}{\phi_1}\left[a_o\left(\frac{\Omega_1}{S^2+\Omega_1^2}\cdot\frac{\phi_1}{S-\phi_1}+\frac{\Omega_2}{S^2+\Omega_2^2}\cdot\frac{\phi_1}{S-\phi_1}\right) \quad -b_o\left(\frac{s}{S^2+\Omega_1^2}\cdot\frac{\phi_1}{S-\phi_1}+\frac{s}{S^2+\Omega_2^2}\cdot\frac{\phi_1}{S-\phi_1}\right)\right] \\ &-\frac{1}{\alpha_2}\left[a_o\left(\frac{\Omega_1}{S^2+\Omega_1^2}\cdot\frac{\phi_2}{S-\phi_2}+\frac{\Omega_2}{S^2+\Omega_2^2}\cdot\frac{\phi_2}{S-\phi_2}\right) \quad -b_o\left(\frac{s}{S^2+\Omega_1^2}\cdot\frac{\phi_2}{S-\phi_2}+\frac{s}{S^2+\Omega_2^2}\cdot\frac{\phi_2}{S-\phi_2}\right)\right]\Bigg\} \end{aligned} \tag{2.52}$$

Where

$$\phi_1 = \frac{-D_2+\sqrt{D_2^2-4D_1D_3}}{2D_1} \qquad and \qquad \phi_2 = \frac{-D_2-\sqrt{D_2^2-4D_1D_3}}{2D_1}$$
$$\Omega_1 = \omega+\frac{k\pi\gamma}{L} \qquad and \qquad \Omega_2 = \omega-\frac{k\pi\gamma}{L} \tag{2.53}$$

Equation (2.52) is analogous to equation (2.44) subjecting equation (2.52) to Laplace transform in conjunction with the boundary condition (2.2) and using convolution theory yields

$$\begin{aligned} y_m(t) = P_p\Bigg\{&\frac{e^{\phi_1 t}}{\phi_1}\Bigg\langle\Bigg[\frac{a_o}{(\Omega_1^2+\phi_1^2)}\left(-\Omega_1 e^{-\phi_1 t}\cos\Omega_1 t+\Omega_1-\phi_1 e^{-\phi_1 t}\sin\Omega_1 t\right) \\ &+\frac{a_o}{(\Omega_2^2+\phi_1^2)}\left(-\Omega_2 e^{-\phi_1 t}\cos\Omega_2 t+\Omega_2-\phi_1 e^{-\phi_1 t}\sin\Omega_2 t\right)\Bigg] \\ &+\Bigg(\frac{b_o}{(\Omega_1^2+\phi_1^2)}\left(-\Omega_1 e^{\phi_1 t}\sin\Omega_1 t+\phi_1-\beta_1 e^{\phi_1 t}\cos\Omega_1 t\right) \\ &-\frac{b_o}{(\Omega_2^2+\phi_1^2)}\left(-\Omega_2 e^{\phi_1 t}\sin\Omega_2 t+\phi_1-\phi_1 e^{\phi_1 t}\cos\Omega_2 t\right)\Bigg)\Bigg\rangle \\ &-\frac{e^{\phi_1 t}}{\phi_2}\Bigg\langle\Bigg[\frac{a_o}{(\Omega_1^2+\phi_2^2)}\left(-\Omega_1 e^{-\phi_2 t}\cos\Omega_1 t+\Omega_1-\phi_2 e^{-\phi_2 t}\sin\Omega_1 t\right) \\ &+\frac{a_o}{(\Omega_2^2+\phi_2^2)}\left(-\Omega_2 e^{-\phi_2 t}\cos\Omega_2 t+\Omega_2-\phi_2 e^{-\phi_2 t}\sin\Omega_2 t\right)\Bigg] \\ &+\Bigg(\frac{b_o}{(\Omega_1^2+\phi_2^2)}\left(-\Omega_1 e^{\phi_2 t}\sin\Omega_1 t+\phi_2-\beta_2 e^{\phi_2 t}\cos\Omega_1 t\right) \\ &-\frac{b_o}{(\Omega_1^2+\phi_2^2)}\left(-\Omega_1 e^{\phi_2 t}\sin\Omega_1 t+\phi_2-\beta_2 e^{\phi_2 t}\cos\Omega_1 t\right)\Bigg)\Bigg\rangle\Bigg\} \end{aligned} \tag{2.54}$$

which on inversion yields

$$\begin{aligned}
w_a(x,t) = \sum_{m=1}^{n} P_p \Bigg\{ \frac{e^{\phi_1 t}}{\phi_1} \Bigg\langle \Bigg[& \frac{a_o}{(\Omega_1^2+\phi_1^2)}\left(-\Omega_1 e^{-\phi_1 t}\cos\Omega_1 t + \Omega_1 - \phi_1 e^{-\phi_1 t}\sin\Omega_1 t\right) \\
& + \frac{a_o}{(\Omega_2^2+\phi_1^2)}\left(-\Omega_2 e^{-\phi_1 t}\cos\Omega_2 t + \Omega_2 - \phi_1 e^{-\phi_1 t}\sin\Omega_2 t\right)\Bigg] \\
& + \Bigg(\frac{b_o}{(\Omega_1^2+\phi_1^2)}\left(-\Omega_1 e^{\phi_1 t}\sin\Omega_1 t + \phi_1 - \beta_1 e^{\phi_1 t}\cos\Omega_1 t\right) \\
& - \frac{b_o}{(\Omega_2^2+\phi_1^2)}\left(-\Omega_2 e^{\phi_1 t}\sin\Omega_2 t + \phi_1 - \phi_1 e^{\phi_1 t}\cos\Omega_2 t\right)\Bigg)\Bigg\rangle \\
- \frac{e^{\phi_1 t}}{\phi_2} \Bigg\langle \Bigg[& \frac{a_o}{(\Omega_1^2+\phi_2^2)}\left(-\Omega_1 e^{-\phi_2 t}\cos\Omega_1 t + \Omega_1 - \phi_2 e^{-\phi_2 t}\sin\Omega_1 t\right) \\
& + \frac{a_o}{(\Omega_2^2+\phi_2^2)}\left(-\Omega_2 e^{-\phi_2 t}\cos\Omega_2 t + \Omega_2 - \phi_2 e^{-\phi_2 t}\sin\Omega_2 t\right)\Bigg] \\
& + \Bigg(\frac{b_o}{(\Omega_1^2+\phi_2^2)}\left(-\Omega_1 e^{\phi_2 t}\sin\Omega_1 t + \phi_2 - \beta_2 e^{\phi_2 t}\cos\Omega_1 t\right) \\
- \frac{b_o}{(\Omega_1^2+\phi_2^2)} & \left(-\Omega_1 e^{\phi_2 t}\sin\Omega_1 t + \phi_2 - \beta_2 e^{\phi_2 t}\cos\Omega_1 t\right)\Bigg)\Bigg\rangle\Bigg\}\sin\frac{m\pi x}{L}
\end{aligned} \tag{2.55}$$

3 RESULTS AND DISCUSSION

3.1 Discussion on the Closed form Solution

The displacement response of an engineering structure under excitation may grow without bound and when this happens it leads to the occurrence called resonance. The effects of this occurrence on dynamical system could be devastating. In particular, it causes cracks, permanent deformation and destruction in structures and makes the structural systems unsaved for its occupants. Thus, it is very pertinent at this juncture to establish the conditions under which this undesirable phenomenon may occur. Equation (2.45) clearly shows that the non-uniform elastic beam resting on elastic foundation will experience resonance effects whenever

$$\beta_1 = \beta_2, \qquad \beta_1^2 = -\gamma^2 \qquad or \qquad \beta_2^2 = -\gamma^2 \tag{3.1}$$

While equation (2.55) shows that the same beam under the action of harmonic variable magnitude moving loads will experience resonance effects whenever

$$\phi_1 = \phi_2, \qquad \phi_1^2 = -\Omega_1^2 \qquad or \qquad \phi_2^2 = -\Omega_1^2 \;\; and \;\; also \;\; \phi_1^2 = -\Omega_2^2 \qquad or \qquad \phi_2^2 = -\Omega_2^2 \tag{3.2}$$

It is also observed that as the foundation modulli increases the critical speed of the dynamical system increases thereby reducing the risk of resonant effects.

3.2 Comments on the Numerical Results

The theory presented in this paper is illustrated numerically. The velocity of the moving load and the length of the beam are respectively v = 8.128m/s and L = 12.192. The values of foundation moduli K are varied between 0 N/m^3 and 4000000 N/m^3. Figs. 1 and 5 display the deflection profile of an elastic beams resting on elastic foundation and subjected to constant and variable magnitude moving load. The figures show that as the value of foundation stiffness K increases the deflection of the beam at various time t decreases. Fig. 2 shows the deflection profile of the simply supported beam under constant magnitude load for various values of axial force and fixed value of foundation stiffness$K = 40000$. It is observed that the higher values of axial force reduce the deflection of the beam. Fig. 6 depicts similar behavior for the transverse displacement of simply supported beam under the action of harmonic variable magnitude loads moving at variable velocity for various values of axial force N. Figs. 3 and 7 display the response amplitudes of simply supported beam respectively to constant and harmonic variable magnitude loads travelling at variable velocity for various values of longitudinal amplitude β and fixed values of foundation stiffness $K = 40000$ and axial force $N = 20000$. The figures clearly show that the response amplitude of the simply supported non uniform beam under the action of both constant and harmonic variable magnitude loads travelling at variable velocity decrease with increase in the values of longitudinal amplitude β. Also Figs. 4 and 8 depict the transverse displacement response amplitude of simply supported Rayleigh beam to constant and harmonic variable magnitude loads travelling at variable velocity for various values of load longitudinal frequency and for fixed values of foundation stiffness $K = 40000$ and axial force $N = 20000$. It is observed from this figure that higher values of the load longitudinal frequency α produce more stabilizing effects on the elastic beam.

Finally, Figs. 9 and 10 depicts the comparison of the response amplitude of a simply supported non uniform Rayleigh beam resting on elastic foundation and subjected to constant and variable magnitude moving loads and exact/numerical comparison for fixed values of axial force N = 40000 and foundation modulus K= 50000.

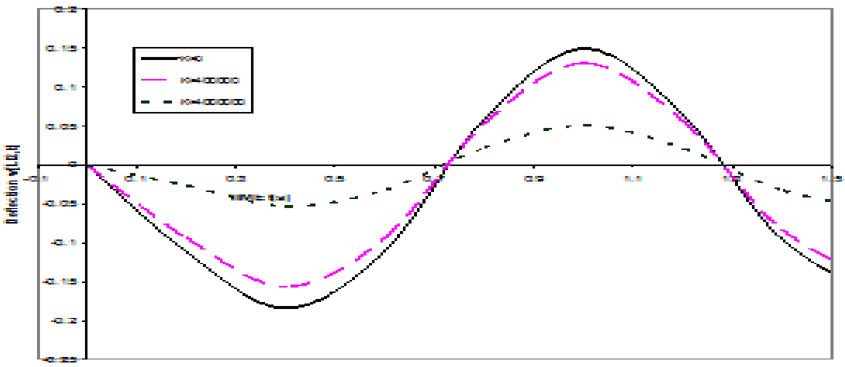

Fig. 1. Deflection profile of a simply supported non- uniform beam under the actions of constant forces travelling at variable velocity for various values of foundation modulus K and fixed value axial force N= 50000

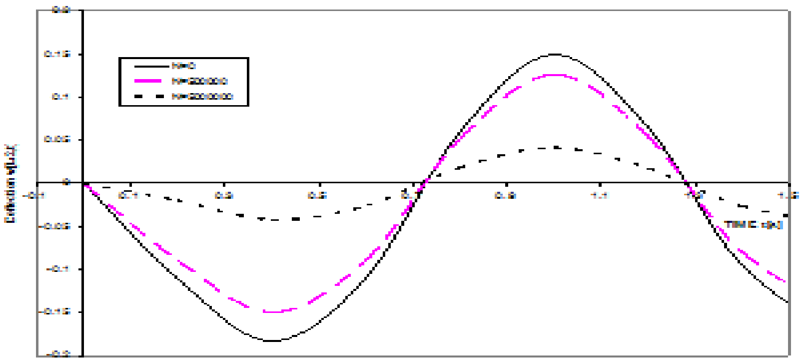

Fig. 2. Response amplitude of a simply supported non uniform beam under the actions of constant forces travelling at variable velocity for various value foundation modulus k=40000

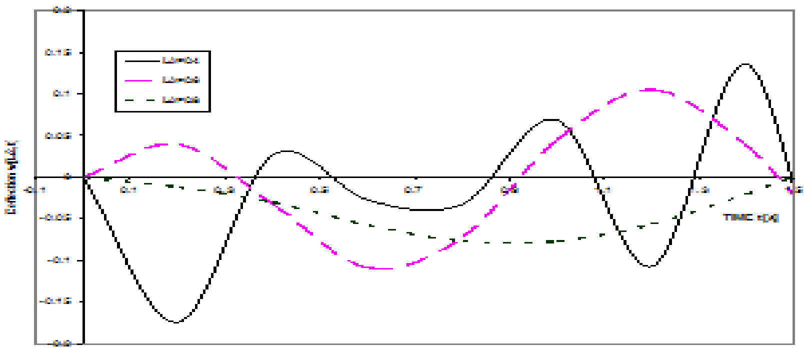

Fig. 3. The response amplitude of a non uniform beam resting on elastic foundation and under the actions of constant magnitude moving load for various values of longitudinal amplitude of oscillation of the load

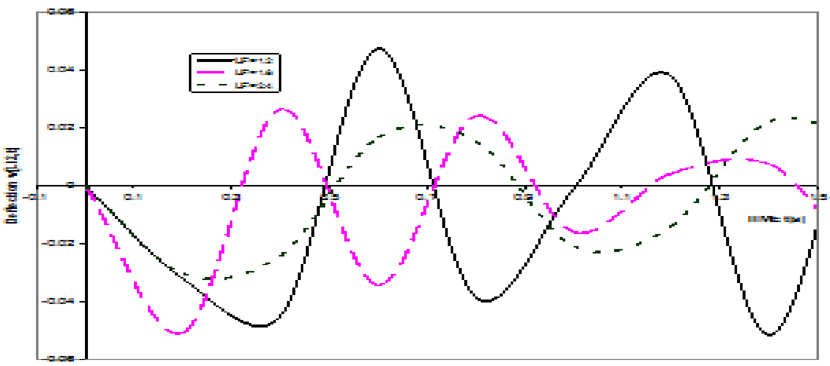

Fig. 4. The displacement response of a non uniform beam resting on elastic foundation and subjected to constant magnitude moving load for various values of longitudinal frequency of the load α

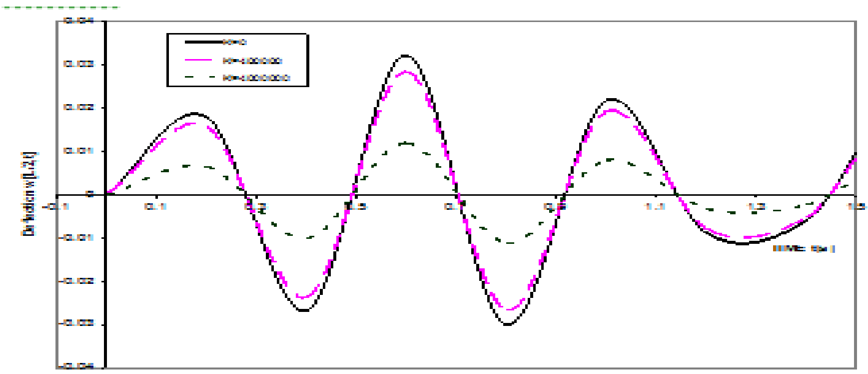

Fig. 5. Response amplitude of a simply supported non uniform beam under the action of harmonic force travelling at variable velocity for various values of foundation modulus and for fixed value axial force N= 40000

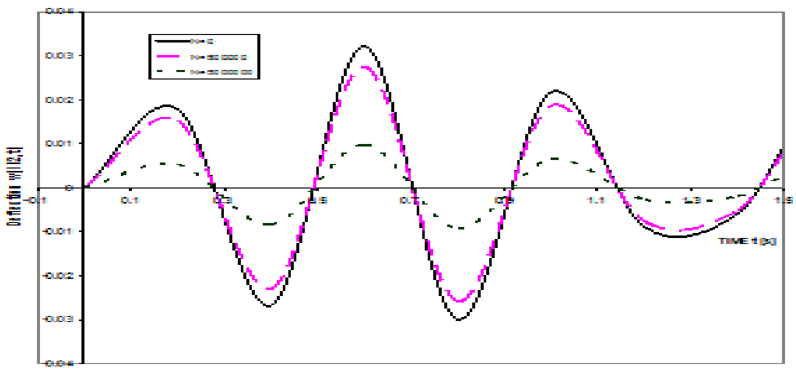

Fig. 6. Response amplitude of a simply supported non uniform beam under the action of harmonic load travelling at variable velocity for various values axial force of and for fixed value foundation modulus K

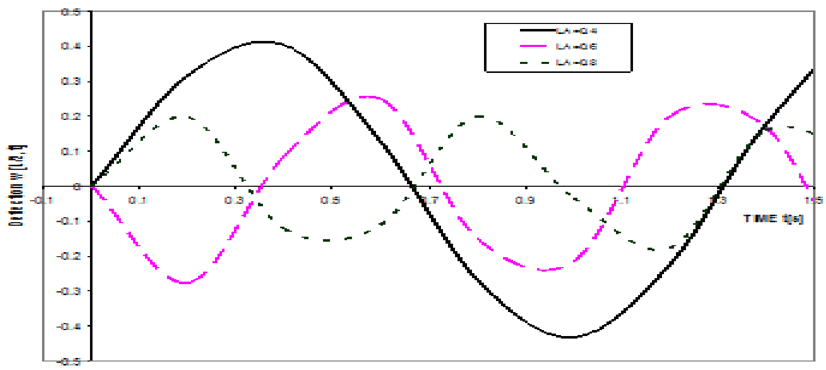

Fig. 7. The response amplitude of a non uniform beam resting on elastic foundation and under the actions of variable magnitude moving load for various values of longitudinal amplitude of oscillation of the load

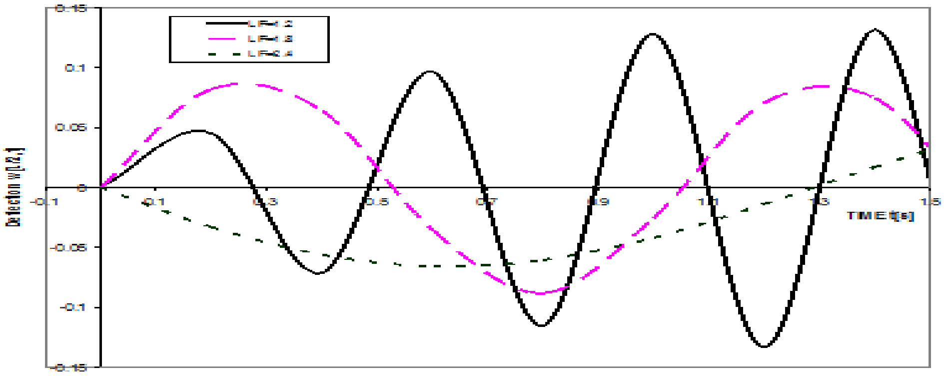

Fig. 8. The displacement response of a non uniform beam resting on elastic foundation and subjected to variable magnitude moving load for various values of longitudinal frequency of the load α

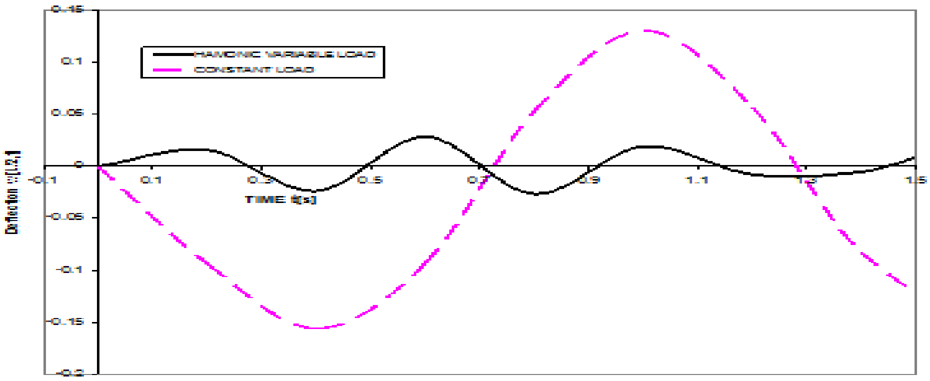

Fig. 9. Comparison of the displacement response of constant load and harmonic load cases of a non uniform simply supported beam for fixed values of K= 40000 and N=40000

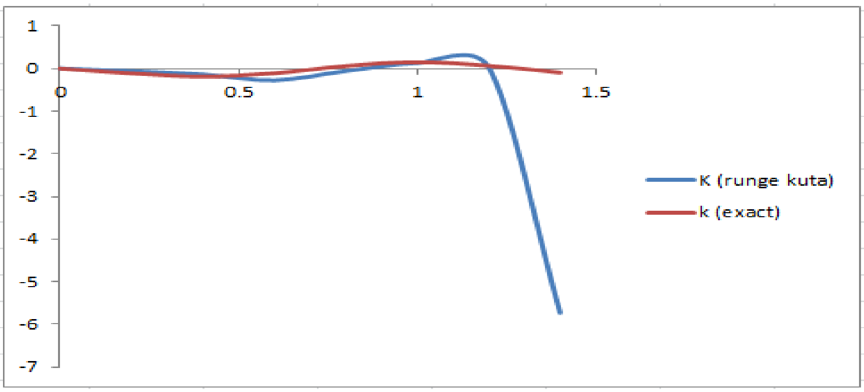

Fig. 10. Comparison of the displacement response of exact solution and numerical solution cases of a non uniform simply supported beam for fixed values of K= 40000 and N=40000

4 CONCLUSIONS

In this paper, a procedure involving the Galerkin's method and integral transform technique has been used to solve the problem of a non-uniform beam when it is subjected to constant and harmonic variable magnitude moving loads. The objective is to study the behavior of the dynamical system. In particular, analytical solution in series form is obtained for the deflection of the elastic beam and the effects of foundation stiffness K and the axial force N on the vibrating system are investigated. Analytical solution and numerical result in plotted curves show that as the value of foundation stiffness K and axial force N increase, the deflection profile of the non-uniform beam decreases. Thus, in general, higher values of foundation stiffness K and axial force N reduce the risk of resonance in a dynamical system involving non-uniform beam under the action of a moving load.

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Competing Interests

Authors have declared that no competing interests exist.

References

[1] Statistic MM, Euler JA, Montgomery ST. On a theory concerning dynamical behavior of structures carrying moving masses. Ing. Archiv. 1974;43:295-305.

[2] Fryba L. Vibration of solids and structures under moving loads. Groningen: Noordhoff; 1972.

[3] Sadiku S, Leipholz HHE. On the dynamic of elastic systems with moving concentrated masses. Ing. Archiv. 1987;57:223-242.

[4] Adedowole A. Flexural motions under moving distributed masses of beam- type structures on vlasov foundation and having time dependent boundary conditions. Ph.D thesis. Federal University of Technology, Akure, Nigeria; 2016.

[5] Oni ST. Flexural vibrations under moving load of isotropic rectangular plates on a non-winkler elastic foundation. Journal of the Nigerian Society of Engineers. 2000;35(1):18-27.

[6] 6. Oni ST, Adedowole A. Influence of prestress on the response to moving loads of rectangular plates incorporating rotatory inertia correction factor. Journal of the Nigerian Association of Mathematical Physics. 2008;13:127-140.

[7] Jimoh SA, Adedowole A. Dynamic response of non-prismatic Bernoulli Euler beam with exponentially varying thickness resting on variable elastic foundation. Asian Research Journal of Mathematical. 2017;6(3):1-15. Available:http://sciencedomain.org/journal/44/articles-press, http://sciencedomain.org/journal/44/articles-press

[8] Lowan AN. On transverse oscillations of beam under the moving variable loads. Phil. Mag. 1935;19(127):708-715.

[9] Kokhmanyuk SS, Filippov AP. Dynamic effects on a beam of a load moving at variable speed. Stroitel in Mekhanka i Raschet So-oruzhenii. 1967;9(2):36-39.

[10] Wang YM. The dynamical analysis of a finite inextensible beam with an attached accelerating mass. International Journal of solid Structure. 1998;35:831-854.

[11] Oni ST, Omolofe B. Dynamic behavior of non-uniform Bernoulli-Euler beams subjected to concentrated loads traveling at varying velocities. Abacus. Journal of Mathematical Association of Nigeria. 2005;32(2A).

[12] Oni ST, Omolofe B. Dynamic response of prestressed Rayleigh beam resting on elastic foundation and subjected to masses travelling at varying velocity. Journal of Vibration and Acoustics (USA). 2011;133:1-15.

[13] Omolofe B, Ogunyebi SN. Dynamic characteristics of a rotating Timoshenko beam subjected to a variable magnitude

load travelling at varying speed. J. KSIAM. 2016;20(1):17-35.

[14] Adedowole A. Flexural vibration of non-prismatic rayleigh beam with non-uniform prestress under concentrated loads moving with variable velocity. Journal of the Nigerian Association of Mathematical Physics. 2017;40:131-142.

[15] Jimoh SA, Oni ST, Ajijola OO. Effect of variable axial force on the deflection of thick beam under distributed moving load. Asian Research Journal of Mathematical. 2017;6(3):1-12.
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