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Forecasting Road Traffic Accidents: Grey System Theory GM(1,1) and Grey Entropy Based Approach (GMEPA)

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Authors' contributions

This work was carried out in collaboration between both authors. Author LA designed the study, performed the statistical analysis, wrote the protocol, managed the literature searches and wrote the first draft of the manuscript. Author SA edited and checked the document. Both authors read and approved the final manuscript.

Article Information

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Case Study

ABSTRACT

Road collision is one of the worst case scenarios involving massive damages and casualties. It has become a major concern for everyday road users as well as the government. Traffic accidents in terms of forecasting can be considered as a grey system considering the complexity and unknown influencing factors causing these accidents. Therefore, it can be analyzed using GM(1,1) since grey model has the criteria of handling limited amount of data to estimate the behavior of an unknown system. However, conventional method of GM(1,1) has several drawbacks that requires improvements in order to provide a more reliable references allowing responsible authorities to come out with strategies to prevent road accidents. In this study, we compare the results of propose method which is the hybrid Grey model with Minimize Entropy Principle Approach (GMEPA) and original grey model GM(1,1) based on the minimization of forecasting error. The

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data used are road traffic accidents in Malaysia from 2003 to 2016 and road traffic accidents in India from year 2002 to 2015. Mean average percentage error(MAPE) and Mean Squared Error(MSE) were calculated for both models to examine which method gives the best prediction accuracy. The results conclude that GMEPA can improve the measurement of forecasting accuracy for road accident data in India but vice versa in road accident data Malaysia where it was much preferable for the application of GM(1,1).

Keywords: Road traffic accident; grey system theory; forecasting; minimize entropy principle Approach (MEPA); GM(1,1).

1. INTRODUCTION

The growth rate of vehicles usage is the backbone of economic development. In a rapidly developing country, road transportation is essential to enable more trade and a greater spread of people. It provides easy access to markets, employment and additional investment. Unfortunately, with the increment of registered vehicles on the road, more accidents happen unexpectedly. Road accidents shows a rising tendency as a result of increasing motor vehicles users. Various factors that cause these car crashes, for instant, self-negligence of road users, vehicles breakdown, weather condition and the imperfection of road infrastructure. Due to the complexity and uncertainty of influencing factors on road traffic accidents, grey system theory is suitable to apply as it fulfills the criteria of overcoming insufficient information when dealing with raw data.

Grey system theory was first introduced by Deng Julong in 1982 [1]. Articles that has been published and accepted widely based on grey system theory are "The control problem of Grey system" and "The Grey Control System" [2]. Grey model forecasting has the specialty of dealing and solving problems under uncertainty [3]. It has the ability of applying only a few set of data with the minimum of 4 [4,5]. The theoretical derivation can be easily understood and the method is simple [6]. It can be applied to various field for example in tourism demand [7], financial and economic [8,9,10] and energy [11].

One of the main challenges is how to forecast trends under the limitation of lack of information while still being able to produce forecast with least prediction errors. Countless approaches have been done by researchers to improve the performance of original GM(1,1). It is important to discover new approach which helps researchers to obtain a much better result compared to conventional GM(1,1) method for forecasting purposes and future planning. It can also provide reliable references to other

authorities in solving problems. To address the issue, a new approach was introduced, which is by combining grey model GM(1,1) with Minimize Entropy Principle Approach(MEPA) where MEPA is a method that function to partition the universe of discourse and build up membership functions(MFs) [12].

The propose method focuses on estimating the parameters of the model to improve the forecasting accuracy. Originally, GM(1,1) uses least square method(OLS) to determine the parameters of "a" and "b" with different α values [13]. While in GMEPA, the values of α is determine in MEPA and will be used to estimate the parameters in GM(1,1). The aim is to reduce the coefficient of "a" because according to Liu and Deng [14], the larger the coefficient "a" will result a larger prediction error.

Firstly, basic concept of GM(1,1) is explained. Then, combined MEPA with GM(1,1) is shown in detailed. Where MEPA is use starting after step 2 in the method of GM(1,1). This study covers the accidents data from year 2003 to 2016 data in Malaysia [15] and traffic collision data in India from year 2002 to 2014 [16].

2. METHODOLOGY

2.1 Grey System Theory GM(1,1)

This model stands for "*single variable first order grey model"* to forecast quantitative data [17]. Grey model uses a understandable formula which includes 3 main operator [18].

- 1. Accumulating Generation Operator (AGO)
- 2. Grey model
- 3. Inverse Accumulating Generation Operator (IAGO)

Step 1: $x^{(0)}$ represent original data

$$
x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))
$$

 $x^{(0)}(i)$ time series on time *I*, n must be equal or more than 4

Step 2: Accumulating Generation Operator(AGO)

A new set $x^{(1)}$ is obtained by using AGO

$$
x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n))
$$

$$
x^{(1)}(t) = \sum_{i=1}^{t} x^{(0)}(t), t = 1, 2, \dots, n.
$$

Step 3: from step 1 and 2, obtained the parameter

Parameter is calculated by traditional least square method

$$
A = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_n
$$

\n
$$
Y_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \quad B = \begin{bmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \vdots \\ -Z^{(1)}(n) & 1 \end{bmatrix} \quad A = \begin{bmatrix} a \\ b \end{bmatrix}
$$

 $Z^{(1)}$ defined as

$$
Z^{(1)} = \{Z^{(1)}(1), Z^{(1)}(2), \dots, Z^{(1)}(i), \dots, Z^{(1)}(n)\}
$$

$$
Z^{(1)}(k) = \alpha x^{(1)}(k) + (1 - \alpha)x^{(1)}(k + 1),
$$

$$
t = 1, 2, ..., n - 1
$$

a symbolized horizontal coefficient and $0.1 < \alpha$ <1.0. The value of α is chosen to obtained the smallest forecast error.

Step 4: theoretically through difference equation, to solve GM(1,1) is as follow:

$$
\hat{x}^{(1)}(t+1) = \left[x^{(0)}(1) - \frac{b}{a}\right]e^{-at} + \frac{b}{a}
$$

Step 5: IAGO is to forecast the value of data at t $+1$

$$
\hat{x}^{(0)}(t+1) = \left[\hat{x}^{(1)}(t+1) - \hat{x}^{(1)}(t) \right]
$$

Where $t = 1, 2, \ldots, n$, sequence stated as below:

$$
\hat{x}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(t+1))
$$

The calculation will be repeated for GM(1,1) using different level of significant which for this paper $\alpha = 0.4, 0.5, 0.6, 0.7$ and 0.8 has been used. Previous research had stated that α =0.5 is the best alpha value for prediction [19]. Additionally, Huang [20] also mentioned in his research that $\alpha = 0.4$ gives a high accuracy towards GM(1,1). Plus, Ho [21] has proved that alpha value that is more than 0.5 can provide the lowest MAPE.

FORECAST:

To obtained forecast values from original data set, the steps in GM(1,1) is repeated.

IAGO calculation is to predict the accidents data at t+1 using:

$$
\hat{x}^{(0)}(t+1) = \left[\hat{x}^{(1)}(t+1) - \hat{x}^{(1)}(t) \right]
$$

Where $t = 1, 2, \ldots, n$, sequence stated as below:

$$
\hat{x}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), ..., \hat{x}^{(0)}(t+1))
$$

n is the future forecast value.

2.2 Minimize Entropy Principle Approach (MEPA)

MEPA is a method that is combined starting after step 2 in Grey model.

Step 1: determining the class of each data entry

Each data has to be assigned a class initially. There is no specific rule to determine the classes of each data and it is due to the character of entropy [12]. But in this paper, a method of Fuzzy C- Mean Clustering [22] is introduced to determine the classes that will be assigned to each data entry from year 1975 to 1995. Previous research by Cheng et al. (2006), 3 classes has been assigned to the sample data. So, in this research also assigned 3 classes towards each data entry.

Step 4: The data set must be sorted (ascending order) based on the value of each year. The threshold value is sought for a sample in the range between x_1 and x_2 . An entropy equation with each value of x is written for the regions $[x_1, x_1 + x]$ and $[x_1 + x, x_2]$ and denote the first part as region p and the second as region q.

Step 5: calculate the threshold value (PRI, SEC1, SEC2, TER1, TER2, TER3, TER4). An entropy with each value of x in the region x_1 and x_2 is expressed as [23].

 $S(x) = p(x) s_p(x) + q(x) s_q(x)$

Where

 $s_n(x) = - [p_1(x) \ln p_1(x) + p_2(x) \ln p_2(x)]$

 $s_a(x) = - [q_1(x) \ln q(x) + q_2(x) \ln q_2(x)]$

 $p_k(x)$ and $q_k(x)$ = conditional probabilities that the class *k* sample is in the region $[x_1, x_1 + x]$ and $[x_1 + x, x_2]$ respectively.

 $p(x)$ and $q(x)$ = probabilities that all sample are in the region $[x_1, x_1 + x]$ and $[x_1 + x, x_2]$ respectively.

 $p(x) + q(x) = 1$

Minimum entropy of x is the optimum threshold value. The value estimates of $p_k(x)$, $q_k(x)$, $p(x)$ and $q(x)$ are calculated as follow:

$$
p_k(x) = \frac{n_k(x) + 1}{n(x) + 1}
$$

$$
q_k(x) = \frac{n_k(x) + 1}{n(x) + 1}
$$

$$
p(x) = n(x)/n
$$

$$
q(x) = 1 - p(x)
$$

Where

 $n_k(x)$ = number of class *k* samples located in $[x_1, x_1 + x]$

 $n(x)$ = the total number of samples located in $[x_1, x_1 + x]$

 $N_k(x)$ = number of class *k* samples located in $[x_1 + x, x_2]$ $N(x)$ = the total number of samples located in $[x_1 + x, x_2]$

 $n =$ total number of samples in [x_1, x_2]

While moving x in the region $[x_1, x_2]$, entropy values is calculated for each position of x , as in Table 1. The value of x in the region that holds the minimum entropy is called the primary threshold (PRI) value. By repeating the process, we can determine the second threshold value denoted by SEC1 and SEC2. Finally, we need to find tertiary threshold values, denoted by TER1, TER2, TER3 and TER4 which complete the seven partitions. Above Table 2 is the calculation of entropy value and the entropy that holds the minimum value $S(x)$ will be the threshold value which is 405876.

Step 6: determine the length of intervals. Calculating standard deviation and let universe of discourse , $U= [D_{min} - \sigma, D_{max} - \sigma]$ Where D_{min} and D_{max} are the minimum and maximum of accidents data in Malaysia.

Standard deviation,
$$
\sigma = \sqrt{\frac{\sum (x - mean x)^2}{N}}
$$

Universe of discourse , $U=[D_{min}-\sigma, D_{max}-\sigma]$ $=$ [298653 – 68842, 521466 + 68842] = [229811 , 590308]

Step 7: calculate each data membership function

$$
\mu_{\hat{A}}(a) = \begin{cases}\n0, & x < a \\
\frac{x-a}{b-a}, & a \leq x < b \\
\frac{c-x}{c-b}, & b \leq x < c \\
0, & x \geq c\n\end{cases}
$$
\n(7.1)

Where

 x represent the accidents data $x^{(0)}(n)$

Year		Data	Classes	Year		Data	Classes
2003		298653		2010	8	414421	2
2004	2	326815		2011	9	449040	3
2005	3	328264		2012	10	462423	3
2006	4	341252		2013	11	477204	3
2007	5	363319	2	2014	12	476196	3
2008	6	373071	2	2015	13	489606	3
2009		397330	っ	2016	14	521466	3

Table 1. Class assigned to data each year

Table 2. Example of entropy value calculation

Table 3. Thresholds of MEPA for Road Accident data in Malaysia

TER1	SEC1	TER2	PRI	TER3	SEC2	TER4
312734	352286	385201	40587 ^r	ヒヒマのの 455 ے ت ㅋvvr	76700	505536

Table 4. Length of intervals of road accident data for Malaysia

The value of a is calculated using equation in (7.1) to obtained a new value of B in Grey model Membership function is calculated using above formula that is later use in step 3 of GM(1,1) to estimate the coefficient of "a" and "b" and followed by the corresponding step to find the prediction value.

3. CASE STUDY

In this section, both GM(1,1) and combined MEPA & GM(1,1) forecasted values are used for comparison. The road traffic accidents data in Malaysia and India is tested to show and detect the accuracy between GM(1,1) and GMEPA(1,1). It is important to evaluate the applicability of the

model by identifying the best model that is suited to the data [24]. The process of data division, according to common opinion $\frac{1}{4}$ of the series will be served as the evaluation(out sample) purposes and the remaining ¾ will be consider as the estimation part(in sample).The data is separated into two section where 2003 to 2012 is the in sample and 2013 to 2016 is the out sample. Whereas for road accident data in India use the data from 2002 to 2011 as in sample and 2012 to 2015 as out sample. The evaluation of accuracy of each method are measured using mean average percentage error(MAPE) and Mean Square Error(MSE).The formulas of MAPE and MSE are shown below:

 $\text{MAPE} = \frac{1}{n} \sum_{t=1}^{n} \frac{|\hat{y}_t - y_t|}{y_t}$ $\frac{n}{t-1} \frac{|y_t - y_t|}{y_t} \times 100$

 \hat{y}_t = the value predicted by the model for time point t

 y_t = the value observed at time point t $n =$ size of the sample

Mean Squared Error, MSE

$$
MSE = \frac{\sum_{t}^{n} e_t^2}{n}, \quad e_t = y_t - \hat{y}_t
$$

 e^2 = error squared

MAPE% **MSE**

 y_t = the value observed at time t (original data) \hat{y}_t = the value predicted by the model at time t

Original and forecasted values are shown in Table 5 and Table 6 to compare both methods accuracy and its mean average percentage error.

The values of parameter "a" and "b" for the road accident data in India is as shown in Table 6. Since the value of parameter a≤0.3, every $alpha(\alpha)$ can be used for long term forecasting purposes [25,26]. Each parameter has increasing pattern with every addition of alpha $(α)$ values. MAPE and MSE that has the lowest result is α=0.8 which is 9% and 2059587405. However, GMEPA is able to improve the measurement of accuracy with MAPE of 8% and MSE of 1840954265. GMEPA also has the capability to estimate the lowest coefficient of "a" and "b" for accident data in India as shown in Table 7.

The values of parameter "a" and "b" for the road accident data in Malaysia is as shown in Table 9. Since the value of parameter a≤0.3, every $alpha(\alpha)$ also can be used for long term forecasting purposes. The parameters shows an increasing pattern with the addition of alpha(α) values. MAPE and MSE that has the lowest result is α=0.4 which is 5% and 820916287 which gives a very highly accurate forecasting based on the interpretation of MAPE values. However, GMEPA is not able to minimize the measurement percentage of MAPE with a result of 8% and MSE of 1804128863. Road Accident in Malaysia shows a positive result towards original GM(1,1).

Table 5. Forecast values and prediction error of the accidents data in India for GM(1,1) with different α values

Year	Original		Estimation Sample						
	data	Forecast Values							
		α = 0.4	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$			
2002	407497	407497	407497	407497	407497	407497			
2003	406726	419844	420891	421944	423001	413934			
2004	429910	430187	431283	432385	433492	424223			
2005	439255	440785	441932	443084	444242	434767			
2006	460920	451644	452843	454049	455260	445574			
2007	479216	462770	464024	465284	466550	456649			
2008	484704	474171	475481	476798	478121	467999			
2009	486384	485852	487221	488596	489978	479632			
2010	499628	497821	499250	500686	502130	491553			
2011	497686	510085	511577	513076	514583	503771			
MAPE%		1	1	1	2	2			
MSE		79925462	78460445	79906440	84306527	127610085			
Evaluation Sample									
2012	490383	522651	524208	525772	527345	516293			
2013	486476	535527	537150	538782	540423	529125			
2014	489400	548720	550413	552115	553826	542277			
2015	501423	562238	564003	565777	567561	555756			

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2666154203 2837690804 3015720613 3200340834 2059587405

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Graph 1. Membership function of MEPA for road traffic accidents in Malaysia

Table 7. Estimation sample and evaluation sample in Road Traffic Accident in India by using concept of MEPA

Evaluation sample

Table 8. Forecast values and prediction error of the accidents data in Malaysia for GM(1,1) with different α values

Evaluation Sample

Table 9. Parameters values for different alpha(α) used in Road Traffic Accident in Malaysia

Table 10. Estimation sample and evaluation sample in Road Traffic Accident in Malaysia by using concept of MEPA

Evaluation sample

Table 11. Interpretation of typical MAPE value

4. CONCLUSIONS AND RESEARCH

In this paper, we compare the accuracy of Grey model and the combining Grey model with Minimize Entropy Principle Approach(MEPA) to forecast road accident data. MEPA is able to

FUTURE estimate a better parameter in GM for road accidents data in India. But Malaysia road accident data performs better in original grey model. This means that only some sample data can used GMEPA(1,1) to improve the measurement of accuracy depending on the estimated coefficient of "a" and "b". For future

studies, researcher can perform GMEPA in other road accident data for other countries or anything that is related to the issue such as road fatalities and road injuries to identify the efficiency of the method.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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