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A Novel Comparison between the Performances of Several Invariant Moments for Printed Multi-Oriented, Multi-Scaled and Noisy Eastern Arabic Numerals Recognition

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Abstract

In this paper, we present a novel comparison between the robustness against noise of Hu, Legendre, pseudo-Zernike and Krawtchouk invariant moments and even more of invariant analytical Fourier-Mellin transform for multi-oriented, multi-scaled and noisy printed Eastern Arabic numerals recognition. These descriptors are used to extract the features from all numeral images. For this purpose in order to pre-process each one of them, we have used the median filter and the thresholding technique for enhancing its quality, while for recognizing each unknown numeral we have exploited the support vectors machine. Furthermore for carrying out efficiently this comparison, we introduce new concepts which are the threshold and the interval of stability of each invariant descriptor and for each Eastern Arabic numeral. The experiments that we have obtained have provided very satisfactory results.

Keywords: *Median filter; thresholding technique; Hu invariant moments; Legendre invariant moments; pseudo Zernike invariant moments; Krawtchouk invariant moments; invariant analytical Fourier-Mellin transform; support vectors machine.*

1 Introduction

Currently, the automated character recognition has increasingly been receiving significant interest and attracted the attention of researchers in the field of pattern recognition. In fact, the great need for fast

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processing by computer of printed and handwritten documents due to the growing amount of produced information makes this domain more important than ever.

In practice, Optical Character Recognition (OCR) is much applied in various domains like printed postal address resolution, bank cheque recognition and signatures recognition, etc.

On the other hand, many studies have been carried out on Latin or Arabic numerals and characters recognition by using moments [1-6] or the support vectors machines [7-10]. In this sense, we are interested in this study to perform a comparison for printed multi-oriented or multi-scaled and noisy Eastern Arabic numerals between the performances of several methods used for extracting the features of numerals which are the Invariant Moments of Hu (HIM) [11], of Legendre (LIM) [12], of Pseudo Zernike (PZIM) [13], of Krawtchouk (KIM) [14], also the Invariant Analytical Fourier-Mellin Transform (IAFMT) [15].

Moreover, each OCR system is in general formed by a set of three principal phases. Firstly, there is a pre-processing used to clean up the character image. Secondly, the features extraction that is exploited for extracting the features or primitives from character and to convert it to a vector. Then, the final phase is the learning-classification used for training all character images of learning database and classifying those of test database. In this context, we have pre-processed all numeral images by the median filter and the thresholding technique and concerning the last phase, we have employed the support vectors machine [16] with the strategy of one against all. In fact, the recognition process that we have opted can be described as follow: after having pre-processed and converted each numeral image to a vector, this last in the learning phase is included in a class labeled by a value equal to 1 while all other vectors for which all other numeral images are transformed are completely grouped in other opposed class takes a label equal to -1, on the other side, we have ten numerals; we will have therefore ten pairs of classes where each of them is separated in an optimal manner by a decision function. Finally in the classification phase, an unknown numeral pre-processed translated, rotated or resized and containing a noise is presented as a vector that its value image is calculated by all ten decision functions already obtained in the learning phase. Then, the recognition will be attributed to numeral that's the decision function which gives the highest value.

2 Recognition System

Our recognition systems are presented in the Fig. 1:

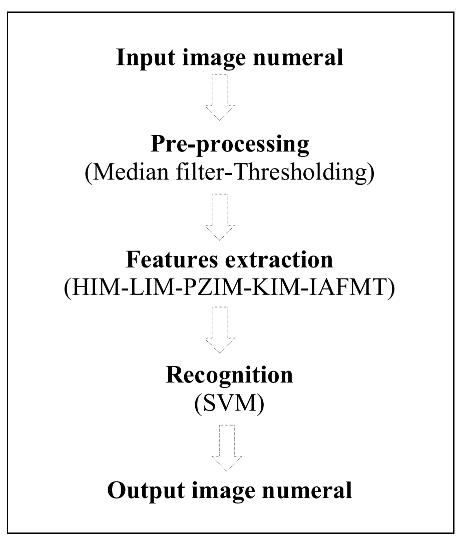

Fig. 1. The proposed recognition systems for Eastern Arabic numerals recognition

3 Pre-Processing

Pre-processing is the first phase of each OCR system, it is used to remove the noise and to suppress the redundant information in order to produce a cleaned up version of each character image in order that it can be used efficiently in the features extraction phase.

In this research, we have pre-processed each numeral image by the median filter used to filtrate it, then by the thresholding (binairization) technique exploited in order to render the image contains only the black and white colors according to a pre-selected threshold (Fig. 2).

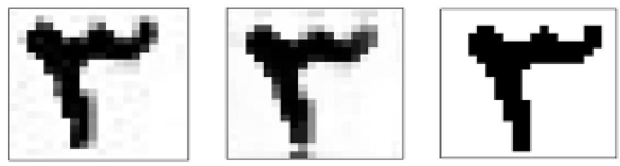

Fig. 2. Pre-processed numeral image

4 Features Extraction

Features extraction is undoubtedly considered as the most important operation in each OCR; furthermore the desired aim of this step is to achieve a great discrimination between different classes of characters by conversion of each character image to a vector in order to render its recognition easier. In this framework, we have used the Hu, Legendre, pseudo Zernike and Krawtchouk invariant moments and invariant analytical Fourier-Mellin transform even more knowing that all these descriptors are very powerful tools exploited for extracting efficiently significant features from characters.

4.1 The Hu Invariant Moments

Firstly, we recall the definitions of the geometric moments of order *(p+q)* of an image function *f(x,y)* having a size *NxM* is:

$$M_{pq} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} x^p y^q f(x, y) \tag{1}$$

These moments are not invariant to geometric transformations: translation, rotation and scaling. For to make it invariant to translation, the central moment of order *(p+q)* is presented:

$$\mu_{pq} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (x - \bar{x})^p (y - \bar{y})^q f(x, y) \tag{2}$$

$\bar{x}$ and $\bar{y}$ are the coordinates of the center of gravity of the image calculated by:

$$\bar{x} = \frac{M_{10}}{M_{00}} \quad \text{and} \quad \bar{y} = \frac{M_{01}}{M_{00}} \tag{3}$$

The centered normalized moment of order *(p+q)* which is at a time invariant to translation and scaling is defined for by:

$$\eta_{pq} = \frac{\mu_{pq}}{m_{00}^{\gamma}}, \gamma = \frac{p+q}{2} + 1, (p+q) \geq 2 \tag{4}$$

Hu was established seven moments following which are invariant to translation, rotation and scaling:

$$\varphi_1 = \eta_{20} + \eta_{02} \tag{5}$$

$$\varphi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \tag{6}$$

$$\varphi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \tag{7}$$

$$\varphi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \tag{8}$$

$$\varphi_5 = (3\eta_{30} - \eta_{21})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \tag{9}$$

$$\varphi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \tag{10}$$

$$\varphi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \tag{11}$$

4.2 The Legendre Invariant Moment

The standard set of the geometric invariant moments that's independent to rotation, scaling and translation is:

$$V_{pq} = M_{00}^{-\gamma} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} [(x - \bar{x})\cos\theta + (y - \bar{y})\sin\theta]^p [(y - \bar{y})\cos\theta (x - \bar{x})\sin\theta]^q f(x, y) \tag{12}$$

where:

$$\bar{x} = \frac{M_{10}}{M_{00}} \text{ and } \bar{y} = \frac{M_{01}}{M_{00}}, \gamma = \frac{p+q}{2} + 1, \theta = \frac{1}{2} arctg \frac{2\mu_{11}}{\mu_{20} - \mu_{02}} \tag{13}$$

And μ_{pq} is the central moment.

The Legendre moments of order *(p+q)* of an image *f(x,y)* can be expressed in terms of geometric moments as shown in equation (14) .In fact, the purpose of V_{pq} (given in equation 12) is to render the Legendre moments also invariant against translation, scaling and rotation:

$$L_{pq} = \frac{(2p+1)(2q+1)}{4} \sum_{i=0}^{p} \sum_{j=0}^{q} a_{pi} a_{qj} M_{ij} \tag{14}$$

a_{pi} is the Legendre polynomial defined by:

$$a_{pi}(x) = P_n(x) = \sum_{i=1}^{n} (-1)^{\frac{n-i}{2}} \frac{1}{2^n} \frac{(n+i)!\,x^i}{(\frac{n-i}{2})!\,(\frac{n+i}{2})!\,i!} \tag{15}$$

$$\text{if} \quad p - q \quad \text{is} \quad \text{even} \quad \text{and} \quad |x| \leq 1 .$$

4.3 The Pseudo-Zernike Invariant Moment

For an image *f(x,y)* the Zernike moment of order n and repetition m is given by:

$$A_{nm} = \frac{n+1}{\pi} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y) V^*(x,y) \tag{16}$$

$$V^*(x,y) = R_{nm}(x,y) e^{im \arctan(y/x)} \tag{17}$$

$$R_{nm}(x,y) = \sum_{s=0}^{n-|m|} \frac{(-1)^s (x^2+y^2)^{\frac{n-s}{2}} (2n+1-s)!}{s!(n-|m|-s)!(n+|m|+1-s)!} \tag{18}$$

$$n \geq |m|, n \geq 0 \ , i = \sqrt{-1} \ , x^2 + y^2 \leq 1$$

the symbol * denotes the complex conjugate operator and t he *NxM* is the number of pixels of an image *f(x,y)*.The pseudo Zernike moment is invariant under rotation but sensitive to translation and scale. So normalization must be done of these moments.

$$f(x,y) = f(\bar{x} + \frac{x}{a} \ , \bar{y} + \frac{y}{a}) \tag{19}$$

where $(\bar{x}, \bar{y})$ is the center of pattern function *f(x, y)*, $a=(\beta/M_{00})^{1/2}$ and β is a predetermined value for the number of object points in pattern.

4.4 The Krawtchouk Invariant Moment

By definition, the Krawtchouk polynomial of order *n* is given by:

$$K_n(x;p,N) = \sum_{k=0}^{N} a_{k,n,p} x^k = {}_2F_1(-n,-x,-N;\frac{1}{p}) \tag{20}$$

$$x, n = 0,1,2 \ldots N \ , N \succ 0 \ , p \in [0,1].$$

${}_2F_1$ is the hyper geometric function defined by:

$$_2F_1(a,b;c;x) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{x^k}{k!} \tag{21}$$

And $(a)_k$ is the pochhammer symbol (rising factorial) defined by:

$$(a)_k = a(a+1)...(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} \tag{22}$$

The Γ function is defined by:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \tag{23}$$

and:

$$\forall n \in N, \Gamma(n+1) = n!$$

the set of *(N+1)* Krawtchouk polynomial $\{k_n(x;p,N)\}$ forms a complete set of discrete basis functions with the weight function:

$$w(x;p,N) = \binom{N}{x} p^x (1-p)^{N-x} \tag{24}$$

and satisfies the orthogonally condition:

$$\sum_{x=0}^{N} w(x;p,N) K_n(x;p,N) K_m(x;p,N) = \rho(n;p,N)\delta_{nm} \tag{25}$$

$$m, n = 0,1,2 \ldots N,$$

$\rho(n;p,N)$ is the squared norm defined by:

$$\rho(n;p,N) = (-1)^n \left(\frac{1-p}{p}\right)^n \frac{n!}{(-N)_n} \tag{26}$$

and δ_{nm} is the Kronecker symbol defined by:

$$\delta_{nm} = \begin{cases} 1 & if \quad n = m \\ 0 & otherwise \end{cases} \tag{27}$$

The Krawtchouk moments have the interesting property of being able to efficiently extract local features of an image this moment of order *(n+m)* of an image *f(x,y)* is given by:

$$Q_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} K_n(x;p_1,N-1) K_m(y;p_2,M-1) f(x,y) \tag{28}$$

the NxM is the size of an image *f(x,y)*.The set of weighted Krawtchouk polynomials $\bar{K}_n(x;p,N)$ is:

$$\bar{K}_n(x;p,N) = K_n(x;p,N)\sqrt{\frac{w(x;p,N)}{\rho(x;p,N)}} \tag{29}$$

the Krawtchouk moment invariant is:

$$\tilde{\Omega}_{nm} = \Omega_{nm}\sum_{i=0}^{n}\sum_{j=0}^{m} a_{i,n,p_1} a_{j,m,p_2} \tilde{V}_{ij} \tag{30}$$

where V_{ij} the invariant geometric moment.

$$\Omega_{nm} = [\rho(n;p_1,N-1).\rho(m;p_2,M-1)]^{-1/2} \tag{31}$$

$$\tilde{V}_{ij} = \sum_{p=0}^{i}\sum_{q=0}^{j}\binom{i}{p}\binom{j}{q}\left(\frac{N^2}{2}\right)^{\frac{p+q}{2}+1}\left(\frac{N}{2}\right)^{i+j-p-q} V_{pq} \tag{32}$$

$$\binom{x}{y} = \frac{x!}{y!(x-y)!} \tag{33}$$

the coefficients $a_{i,n,p}$ are determined in equation (20).

4.5 The Invariant Analytical Fourier-Mellin Transform

The standard Analytical Fourier-Mellin Transform (AFMT) of a function *f(r,θ)* in polar coordinates is given by:

$$M_{f_s}(k,v) = \frac{1}{2\pi}\int_0^{\infty}\int_0^{2\pi} f(r,\theta) r^{s-jv} e^{-ik\theta} d\theta \frac{dr}{r}$$
$$\forall (k,v) \in \mathbb{Z}\times\mathbb{R} \quad , i = \sqrt{-1} \ , s \succ 0 \tag{34}$$

the Invariant Analytical Fourier-Mellin Transform (IAFMT) to translation, rotation and scale of an function *f(r,θ)* is defined by :

$$I_{f_\sigma}(k,v) = M_{f_\sigma}(0,0)^{\frac{-s+jv}{s}} e^{ik\arg(M_{f_\sigma}(1,0))} M_{f_s}(k,v) \tag{35}$$

the AFMT can be written in a Cartesian coordinates as following:

$$M_{f_\sigma}(k,v) = \frac{1}{2\pi}\sum_{x=0}^{M-1}\sum_{y=0}^{M-1} f(x,y)(x+iy)^{-k}(x^2+y^2)^{\frac{k-2+s-iv}{2}} \tag{36}$$

the *NxM* is the number of pixels of an image *f(x,y)*.

5 Recognition

5.1 The Support Vectors Machine

The goal of the SVM is to find a classifier that separates between both classes and maximizes as much the distance between them knowing that the first class contains a set of vectors which are labeled by the value 1 and the second class is formed by other vectors that bears a label equal to -1 (See Fig. 3).

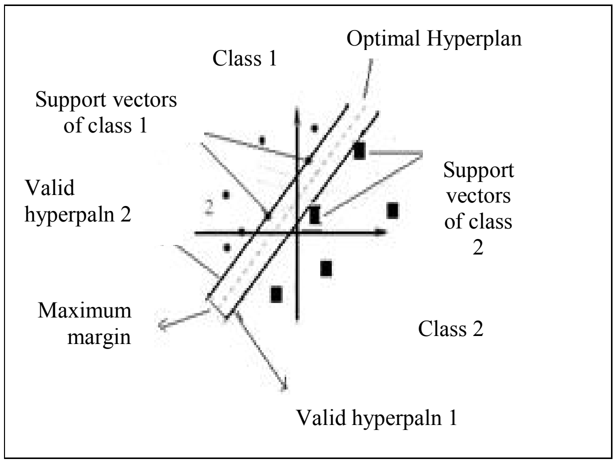

Fig. 3. Linear support vectors machines illustration.

In practice, the nearest vectors separating both classes are only used for finding this classifier and are called the support vectors. However the property of SVM is that this classifier must be optimal, in other words it must to maximize as possible the distance between support vectors of a class and those of other opposite class. The classifier is represented by a special function named a decision function:

$$f(x, w, b) : x \longrightarrow y \qquad (37)$$

where $x \in R^n$, $y \in \{-1, 1\}$ w and b are the parameters of the classifier and y is the label.

For maximizing the distance between the support vectors of a class and those of other class, it must to solve a minimization problem under constraints called the primal problem:

$$\text{To minimize } \frac{1}{2}\|w\|^2 \qquad (38)$$

Subject to $y_i(wx_i + b) \geq 1$, $\forall i = 1,2,.....,n$

the Lagrangian operator of the primal problem is:

$$L(w,b,\alpha) = \frac{1}{2}\|w\|^2 - \sum_{i=1}^{n} \alpha_i [y_i(wx_i + b) - 1] \qquad (39)$$

the variables α_i are called Lagrange multipliers. The associated dual problem is:

To maximize $D(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j K(x_i, x_j)$ (40)

Subject to $\sum_{i=1}^{n} \alpha_i y_i = 0$, $0 \le \alpha_i \le C \quad \forall i = 1,2,.....,n$

where C is a positive constant fixed in advance called constant of penalty. Only the α_i* corresponding to the support vectors is non-zero, the decision function is given by:

$$f(x) = \sum_{i=1}^{n} \alpha_i^* y_i K(x_i, x) + b \tag{41}$$

Where K is called kernel function. In this sense some kernel functions used in the nonlinear SVM are presented in Table 1.

Table 1. Examples of different kernel functions used in SVM

Kernel linear	xy
Polynomial kernel of degree n	$(axy + b)^n$
Gaussian radial basis kernel function (GRBF) with standard deviation σ	$e^{-\frac{\|x-y\|^2}{2\sigma^2}}$

6 Experiments and Results

The desired aim from this study is to compare between the performances of all these descriptors cited previously, more precisely the performances means to three characteristics which are:

- The precision means to recognize correctly a certain numeral translated, rotated or rescaled but not noisy.
- The rapidity means to recognize in a short time a certain numeral.
- The stability means to recognize correctly a certain numeral but in presence of noise.

For this purpose, in our study, we used the following data:

- Each numeral image has a size equal to 40x40 pixels.
- The Krawtchouk parameters p=0.95, q=0.45.
- The parameter of Fourier-Mellin is equal to s=1.
- The kernel function used in the SVM is that of GRBF with a standard deviation equal to 0.75 and a constant of penalty equal to 10^6.
- The different angles which each numeral is rotated are : -360°,-270°,-180°,-90°,0° ,90°,180°, 270º, 360º.

Firstly, we present a test numeral in different situations: translated, rotated or resized and not noisy, then we add progressively a quantity of noise of type 'Gaussian' for knowing its effect on the rate recognition of each numeral and to global rate also. The standard deviation values of Gaussian noise are: [0, 0.01,……, 0.29, 0.30] while that its mean value is fixed to $\mu = 0.05$.

In this context, some examples of numerals of test are given in Fig. 4.

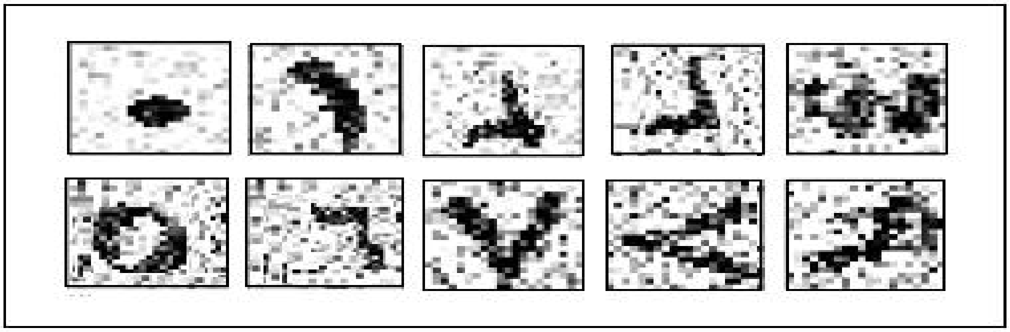

Fig. 4. Eastern Arabic numerals rotated, translated and rescaled and noisy for different values of the parameter standard deviation of Gaussian noise.

We group the values of the recognition rate τ_n (given in %) and the time of execution (given in second) of each numeral N that we obtained for each descriptor in the Table 2:

Table 2. The recognition rate of each numeral for each descriptor

N	τ_n(HIM)	τ_n (LIM)	τ_n (PZIM)	τ_n (KIM)	τ_n (IAFMT)
٠	93.55	100.0	100.0	100.0	100.0
١	87.10	93.55	100.0	100.0	100.0
٢	83.87	87.10	90.32	100.0	93.55
٣	74.19	83.87	87.10	93.55	90.32
٤	61.29	70.97	83.87	90.32	83.87
٥	80.65	87.10	90.32	93.55	90.32
٦	83.87	90.32	90.32	100.0	100.0
٧	58.06	58.06	70.97	83.87	74.19
٨	67.74	74.19	74.19	90.32	87.10
٩	58.06	61.29	70.97	80.65	70.97
t_g	**220,25**	**396,84**	**318,63**	**437,78**	**385,41**

The associated graphical representation to Table 2 is:

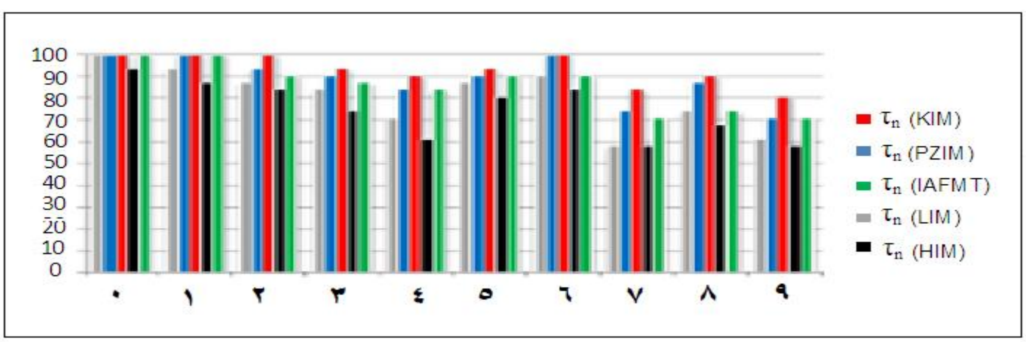

Fig. 5. Graphical representation of recognition rate of each numeral for each descriptor

Furthermore, we present the recognition time i.e. time of execution (given in second) of each numeral and for each descriptor in Fig. 6:

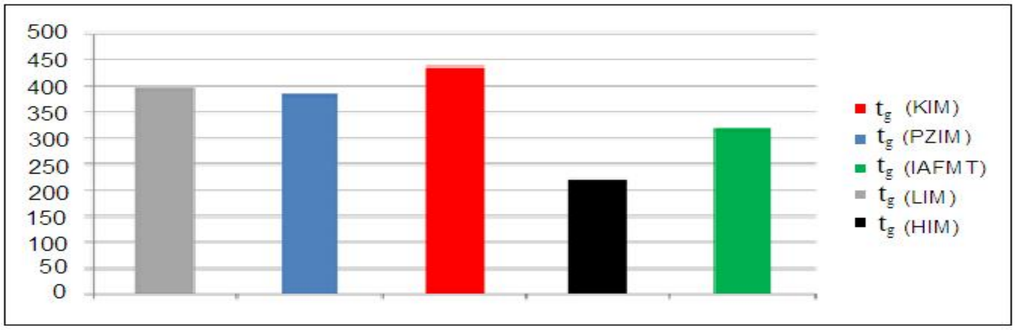

Fig. 6. Graphical representation of global time of execution for each descriptor

We present also the variation of global recognition rate τ_g (given in %) i.e. of all numerals depending upon added noise in Table 3 and its associated graphical representation in Fig. 7:

Table 3. The global recognition rate τ_g depending upon added noise of each descriptor

Noise	τ_g (HIM)	τ_g (LIM)	τ_g (PZIM)	τ_g (KIM)	τ_g (IAFMT)
0.00	100	100	100	100	100
0.01	100	100	100	100	100
0.02	100	100	100	100	100
0.03	100	100	100	100	100
0.04	100	100	100	100	100
0.05	100	100	100	100	100
0.06	100	100	100	100	100
0.07	100	100	100	100	100
0.08	100	100	100	100	100
0.09	100	100	100	100	100
0.10	100	100	100	100	100
0.11	100	100	100	100	100
0.12	100	100	100	100	100
0.13	100	100	100	100	100
0.14	100	100	100	100	100
0.15	100	100	100	100	100
0.16	100	100	100	100	100
0.17	80	90	100	100	100
0.18	70	80	100	100	100
0.19	70	80	100	100	100
0.20	60	80	100	100	100
0.21	60	70	90	100	80
0.22	50	60	80	100	70
0.23	50	60	80	100	70
0.24	40	60	80	90	70
0.25	40	50	70	80	60
0.26	20	30	60	80	50
0.27	10	20	40	60	20
0.28	10	10	30	40	20
0.29	0	10	30	40	20
0.30	0	10	30	40	20

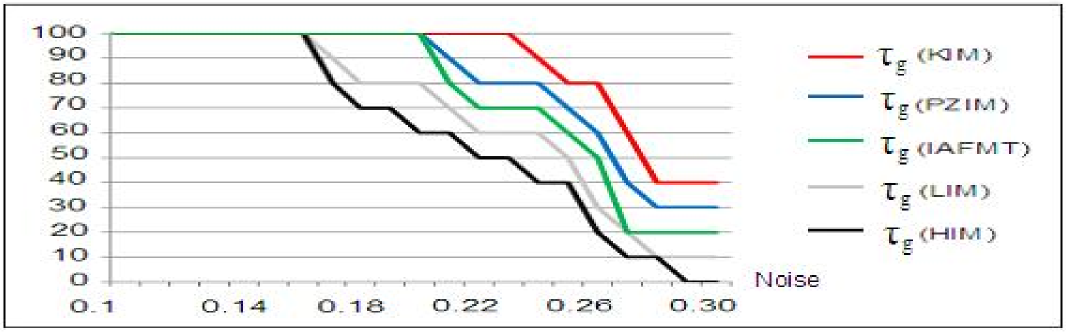

Fig. 7. Graphical representation of global rate recognition depending upon added noise for each descriptor

Analysis and Comments:

Taking into account the results that we have obtained in Tables 2 and 3, Figs. 5, 6 and 7, we really concluded that:

- The most precise and stable descriptor is the KIM then PZIM followed by IAFMT then LIM finally HIM.
- The fastest descriptor is HIM then IAFMT then PZIM followed by LIM afterwards finally KIM.
- For each descriptor the global recognition rate is in general a decreasing function according to noise added to each numeral. we speak this time of an enforced falling of stability of each recognition system due to presence of noise.

6.1 Recognition Thresholding and Recognition Interval

Now, we propose for each numeral to classify all these descriptors according to degree of robustness (degree of stability) of each one of them against noise presented in this numeral. For this purpose, we note f_i the decision function that separates the class which is labeled by 1 of a numeral N_i to other opposite class which is labeled by -1 contains the rest of all other numerals N_j for i , j = { ٠,١,٢,٣,٤,٥,٦,٧,٨,٩} $i \neq j$ .The class of a test numeral (unknown numeral) $N_{test,j}$ is:

$$class(N_{test,i}) = \arg(\max_i (f_0(N_{test,i})...f_j(N_{test,i})...f_9(N_{test,i})) \quad (42)$$

As we already said, every numeral of test $N_{test,i}$ translated, rotated, or resized and not noisy is correctly recognized, that is to say the largest value of all the values images of the vector modelling $N_{test,I}$ by all the decision functions is f_i , $\forall$ i= { ٠,١,٢,٣,٤,٥,٦,٧,٨,٩}.

For example, the decision function f_j that separates optimally the numeral N_j to other numeral N٠ .... N_{j-1} , N_{j+1} ..... N٩ if the test numeral $N_{test,j}$ is not much noisy, then it's correctly recognized in other words:

$$\arg(\max_j (f_0(N_{test,j})...f_j(N_{test,j})...f_9(N_{test,j})) = j \quad (43)$$

But the equation above is it just up to what amount of added noise to $N_{test,j}$?

Practically, we found that for every test numeral $N_{test,j}$ $\forall$ j= { ٠,١,٢,٣,٤,٥,٦,٧,٨,٩} there is a noise value added to it from where the f_j is no more maximum among all the decision functions $f_٠, f_١, \ldots f_٩$. This noise value is called the threshold of recognition or of tolerance or of stability $n_{t,j}$.

In other word it's the amount of noise from which the numeral $N_{test,j}$ will be badly recognized.

Therefore, into interval [0 $n_{t,j}$], we say in this once that the recognition system is a stable, and is instable elsewhere.

For the determination of the threshold of stability $n_{t,j}$ for each numeral $N_{test,j}$, we must follow the evolution of f_j in function of added noise to $N_{test,j}$.

Therefore we regroup the approximate variations of decision function f_j in function of noise n є [0;0.30] for five descriptors in one graph of each numeral j = { ٠,١,٢,٣,٤,٥,٦,٧,٨,٩} (we restrict to some values of noise which are for example { 0, 0.1, 0.2, 0.3 } and furthermore we neglect a very few points of instability), More precisely, each one of the following figures (Figs. 8, 9, 10-18) presents for each numeral j = { ٠,١,٢,٣,٤,٥,٦,٧,٨,٩} a graph contains the evolution of five f_j where each one of them is associated to a descriptor:

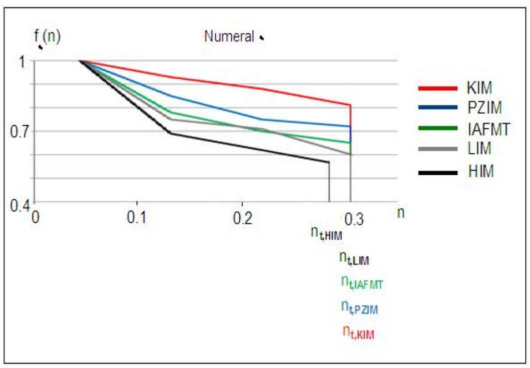

Fig. 8. The variation of decision function f٠ according to added noise to numeral ٠ for five descriptors

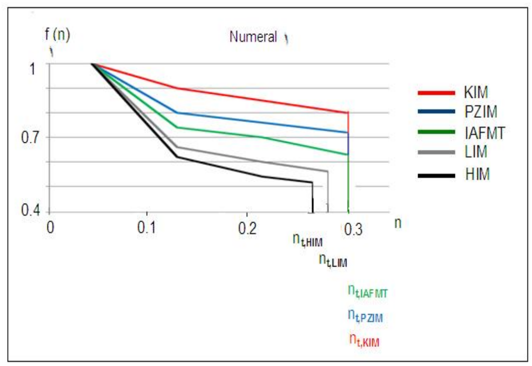

Fig. 9. The variation of decision function f١ according to added noise to numeral ١ for five descriptors

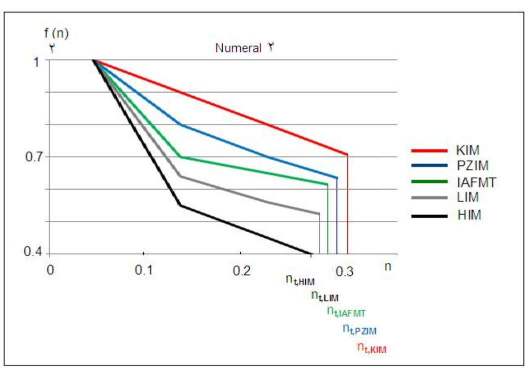

Fig. 10. The variation of decision function f٢ according to added noise to numeral ٢ for five descriptors

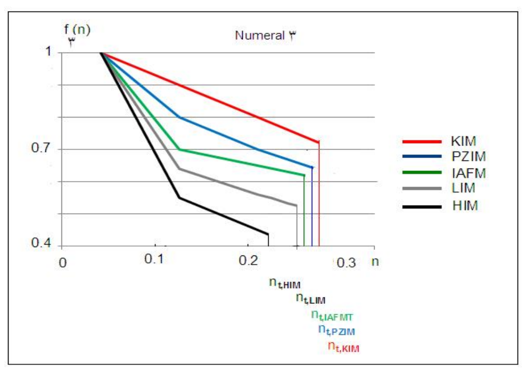

Fig. 11. The variation of decision function f٣ according to added noise to numeral ٣ for five descriptors

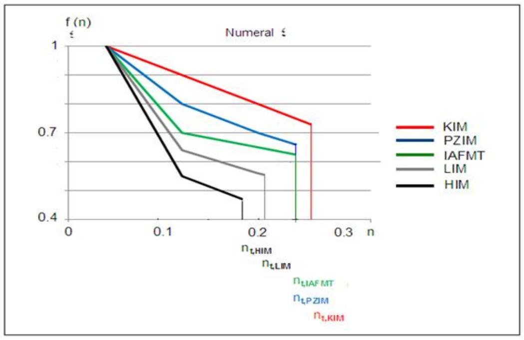

Fig. 12. The variation of decision function $f_{٤}$ according to added noise to numeral ٤ for five descriptors

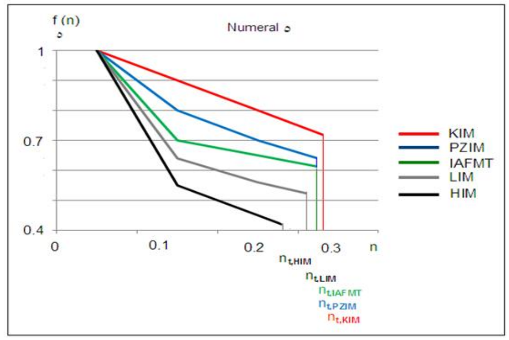

Fig. 13. The variation of decision function $f_{٥}$ according to added noise to numeral ٥ for five descriptors

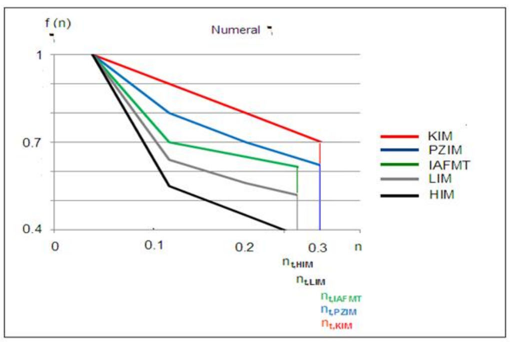

Fig. 14. The variation of decision function $f_{٦}$ according to added noise to numeral ٦ for five descriptors

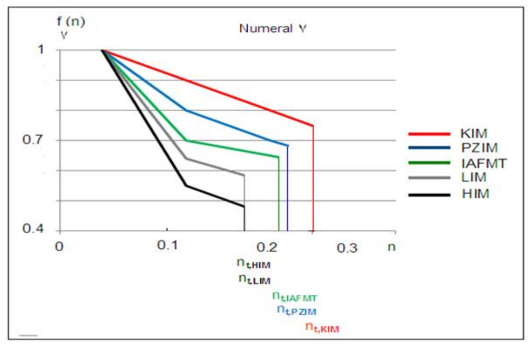

Fig. 15. The variation of decision function $f_{٧}$ according to added noise to numeral ٧ for five descriptors

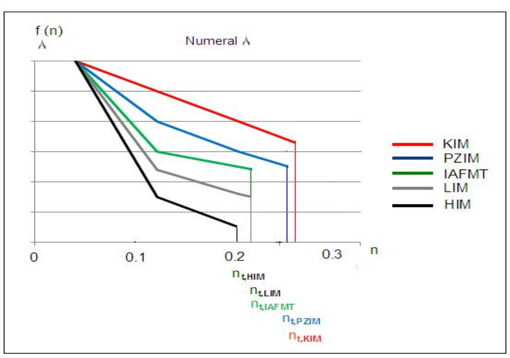

Fig. 16. The variation of decision function $f_{٨}$ according to added noise to numeral ٨ for five descriptors

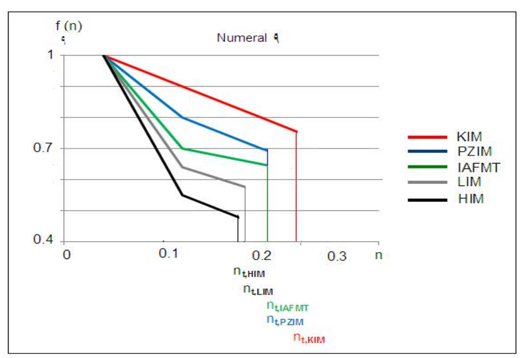

Fig. 17. The variation of decision function $f_{٩}$ according to added noise to numeral ٩ for five descriptors

Analysis and Comments:

Considering these results we conclude that:

- For the five descriptors, the decision function of each numeral is generally a decreasing function according to added noise and that the decreasing of the decision function associated to KIM is smaller than that associated to PZIM which is smaller than that of IAFMT which is smaller than that of LIM which is smaller than that of HIM.
- The most great threshold of recognition or of stability or of tolerance of KIM for all numerals is generally that of PZIM followed by that of IAFMT followed by that of LIM followed by that of HIM, same conclusion for the stability interval (SI). For each *i*= { ٠,١,٢,٣,٤,٥,٦,٧,٨,٩}

Of viewpoint of classification the straight line $f(n_j) = f(n_{t,j})$ represents the optimal hyperplane of separation between the noisy numeral j and the other numerals ٠,١…..j-1, j+1,….. ٩, that is to say for a test numeral $N_{test,j}$ for j = { ٠,١,٢,٣,٤,٥,٦,٧,٨,٩}:

$\forall n_j \leq n_{j,t}$ (Interval or zone of stability of numeral j) we have:

$$\arg(\max(f_{٠}(N_{test,j}), f_1(N_{test,j}).....f_j(N_{test,j}).....f_{٩}(N_{test,j})) = j \tag{44}$$

or:

$$f_j(n_j) \geq f(n_{j,t}) \tag{45}$$

That is to say, this numeral $N_{test,j}$ will be correctly recognized. On the other hand, we call:

$$\Delta f_j = f_j(n_j) - f_j(0) \tag{46}$$

the nearing (migration due to presence of noise) of numeral $N_{test,j}$ to the decision boundary which is nothing other than the optimal separating hyperplane:

$$f(n_j) = f(n_{j,t}) \tag{47}$$

$n_{j,t}$ is the critical value of stability (threshold of stability of numeral j).

$$\Delta f_{j,t} = f_j(n_{j,t}) - f_j(0) \tag{48}$$

is the maximum tolerated falling of the decision function f_j so that the numeral $N_{test,j}$ is correctly recognized.

We grouped all obtained values of the threshold of stability for each numeral in Table 4:

For each numeral, its stability interval is obtained by replacing the $n_{t,j}$ by its numeric value in Table 3 in the interval $[0 \quad n_{t,j}]$.

We take for example the numeral ٣.

Table 4. The different values of threshold of stability for each numeral for five moments

N	$n_{t,HIM}$	$n_{t,LIM}$	$n_{t,PZIM}$	$n_{t,KIM}$	$n_{t,IAFMT}$
٠	0.28	0.30	0.30	0.30	0.30
١	0.26	0.28	0.30	0.30	0.30
٢	0.25	0.26	0.28	0.30	0.27
٣	0.22	0.25	0.27	0.28	0.26
٤	0.18	0.21	0.25	0.27	0.25
٥	0.24	0.26	0.27	0.28	0.27
٦	0.25	0.27	0.30	0.30	0.27
٧	0.17	0.17	0.22	0.25	0.21
٨	0.20	0.22	0.26	0.27	0.22
٩	0.17	0.18	0.21	0.24	0.21

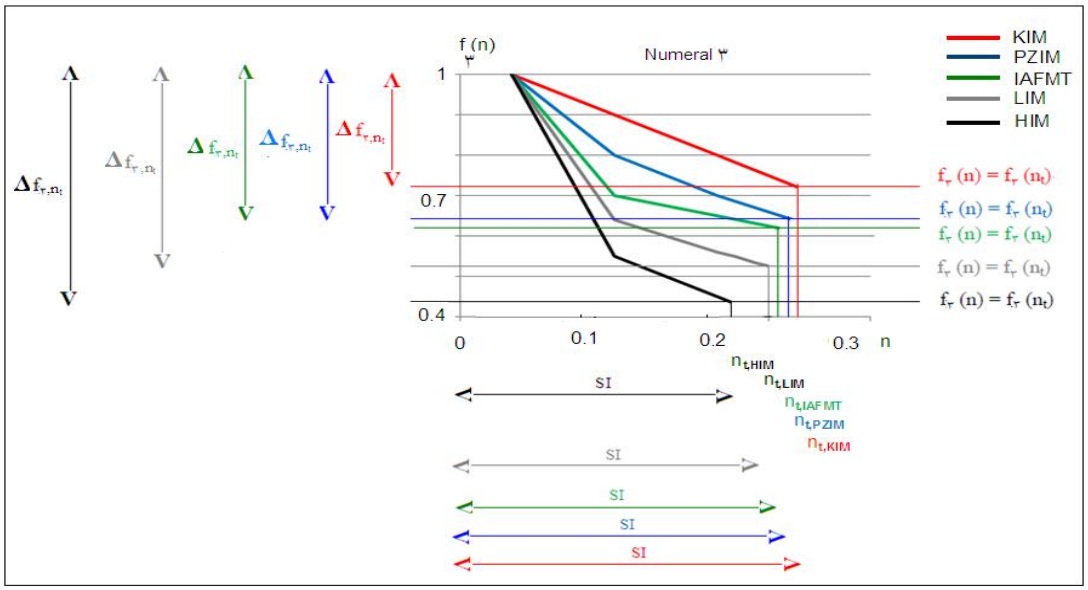

Fig. 18. The determination of threshold stability, optimal hyperplane and stability interval of numeral ٣ for five moments

Analysis and comments:

- The largest interval of stability is that of KIM followed by that of PZIM followed by IAFMT then LIM finally HIM.
- The falling of the decision function according to added noise of the KIM is smallest than that of PZIM which is smallest than that of IAFMT which is smallest than that of LIM which is smallest than that of HIM.

We present some values of the nearing $\Delta f_{٣}$ to the optimal hyperplane (boundary of decision) of the numeral ٣ associated to some values of noise in Table 5:

Table 5. The maximum tolerated falling of the decision function for each numeral for the five moments

Noise	HIM	LIM	IAFMT	PZIM	KIM
	$\Delta f_{۲}$	$\Delta f_{۲}$	$\Delta f_{۲}$	$\Delta f_{۲}$	$\Delta f_{۲}$
0.10	0.35	0.22	0.20	0.16	0.10
0.15	0.42	0.26	0.25	0.20	0.13
0.20	0.50	0.32	0.28	0.24	0.17
$n_{۲,t}$	0.56	0.48	0.38	0.34	0.28

- The falling of decision function or nearing towards the optimal hyperplane (boundary of decision) of HIM is greater than that of LIM which is greater than that of IAFMT which is greater than that of PZIM which is greater than that of KIM, which shows that most stable descriptor is KIM the PZIM then IAFMT, then LIM then HIM.
- More than noise increases more than a numeral j approaches to the boundary of decision, and if $\forall n_j \leq n_{j,t}$ this numeral will be correctly recognized. And if $\forall n_j \geq n_{j,t}$ this numeral will be badly recognized.

We denote:

For n=0 (there will be no noise, i.e. each numeral is multi-oriented and multi-scaled numeral but not noisy)

$$\Delta P = \tau_{g,D_1} - \tau_{g,D_2} \tag{49}$$

the difference of precision between both descriptor D_1 and D_2.

For $n > 0$:

$$\Delta S(n) = \tau_{g,D_1}(n) - \tau_{g,D_2}(n) \tag{50}$$

the difference of stability between both descriptor D_1 and D_2 for a value noise n.

Therefore:

- If $\Delta S(n)>0$ we will have a gain of stability, in this case $\Delta S(n)$ is called the rate of growth of stability.
- If $\Delta S(n)<0$ we will have a losing of stability, in this case $\Delta S(n)$ is called the rate of decay of stability.

$$\Delta R(n) = t_{g,D_1}(n) - t_{g,D_2}(n) \tag{51}$$

the difference of rapidity between both descriptor D_1 and D_2 for a value noise n.

Therefore:

- If $\Delta R(n)>0$ we will have a advancement of rapidity, in this case $\Delta R(n)$ is called the rate of decay of rapidity.
- If $\Delta R(n)<0$ we will have a delay of rapidity, in this case $\Delta R(n)$ is called the rate of growth of rapidity.

We regroup the values of difference of stability and its associated difference of rapidity between KIM and each one of HIM, LIM, PZIM and IAFMT in Table 6:

Table 6. The difference of stability and of rapidity between KIM and HIM, LIM, PZIM and IAFMT

Noise		**0.00**	**0.10**	**0.15**	**0.20**	**0.25**	**0.30**
KIM-HIM	ΔS (%)	0	0	0	40	40	40
	ΔR(second)	27.11	27.95	28.32	29.05	29.72	32.55
KIM-LIM	ΔS (%)	0	0	0	20	30	30
	ΔR(second)	18.14	18.65	18.12	18.74	19.20	20.85
KIM-PZIM	ΔS (%)	0	0	0	0	10	10
	ΔR(second)	13.87	14.15	14.61	15.03	15.56	15.96
KIM-IAFMT	ΔS (%)	0	0	0	0	20	20
	ΔR(second)	22.25	22.87	22.32	22.97	22.54	22.98

Analysis and comments:

Considering the results obtained in table above, we can conclude that:

- Into the interval of noise [0 0.15] the most performing descriptor is HIM due to its high precision and its stability also its rapidity.
- Into the interval of noise [0.15 0.20] the most performing descriptor is IAFMT due to its high precision and its stability also its rapidity.
- Into the interval of noise [0.20 0.30] the most performing descriptor is KIM due to its high precision and its stability but it is slowest.

For example n=0.30 when we replaced HIM by:

- KIM we will have a gain in stability by 40% but in constrast we will have lateness equal to 32.55 second.
- LIM we will have a gain in stability by 10% but in contrast we will have lateness equal to 11.70 second.
- PZIM we will have a gain in stability by 30% but in constrast we will have lateness equal to 16.59 second.
- IAFMT we will have a gain in stability by 20% but in constrast we will have lateness equal to 10.57 second.

Therefore in order to win a certain rate of stability we are really loss a certain rate of rapidity.

- **Probabilistic modelization**

We denote C_j the class of numeral j for j= {٠, ١….٩}, and:

$$P\left(C_j / N_{test,i}\right) \tag{52}$$

is the probability of recognition of the numeral j knowing that $N_{test,i}$, therefore the probability matrix of recognition is:

$$P = P_{ij} = \begin{bmatrix} P(C_٠/٠) & P(C_١/٠) \cdots\cdots P(C_٩/٠) \\ P(C_٠/١) & P(C_١/١) \quad\quad P(C_٩/١) \\ \vdots & \vdots \quad \ddots \quad \vdots \\ P(C_٠/٩) & P(C_١/٩) \cdots\cdots P(C_٩/٩) \end{bmatrix} \tag{53}$$

The set of numerals is *N*= { ٠,١,٢,٣,٤,٥,٦,٧,٨,٩}.Seen all that we have obtained, we write:

$\forall n_j \leq n_{j,t}$ (i.e. into the interval of stability of numeral j) we have the four following results:

Result 1:

$$P(C_j / N_{test,i}) = \delta_{ij} = \begin{cases} 1 & if \quad n = m \\ 0 & otherwise \end{cases} \tag{54}$$

Where δ_{ij} is the Kronecker symbol.

We are saying therefore that this probabilistic law converges to a deterministic law.

Result 2:

The probability matrix recognition converges to identity matrix of order 10.

$$P = I_{10} = \begin{pmatrix} 1 & 0 & \cdots\cdots & 0 \\ 0 & 1 & \cdots\cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots\cdots & 1 \end{pmatrix} \tag{55}$$

Result 3:

The matrix of confusion is:

$$M_C = P = I_{10} \tag{56}$$

Result 4:

The rate recognition of a numeral j is

$$\tau_{n,j} = P_{jj} = 1 = 100\% \tag{57}$$

The global rate recognition i.e. of all numerals is :

$$\tau_g = trace(P_{jj}) = \frac{\sum_{j=٠}^{٩} P_{jj}}{card(N)} = \frac{10}{10} = 1 = 100\% \tag{58}$$

7 Conclusion

The obtained results in the recognition of noisy printed multi-oriented, multi-scaled and noisy Eastern Arabic numerals show that reliable recognition is possible by using the median filter and thresholding technique in the pre-processing phase, the Hu, the Legendre, the pseudo Zernike and Krawtchouk invariant moments also invariant analytical Fourier-Mellin transform in the features extraction phase and the support vectors machine in the recognition phase. The desired goal in this research is to compare between the performances in terms of precision, rapidity and stability all these invariant descriptors. For this purpose in order to achieve efficiently this comparison; we have introduced new concepts which are the threshold and interval of stability and the optimal straight line of each noisy numeral and for each descriptor, also we have introduced the difference between precision, rapidity and stability between each both descriptor. The results that we have obtained verified undoubtedly that the most stable descriptor is that of Krawtchouk followed by pseudo Zernike then Fourier-Mellin then Legendre then finally followed by Hu.

On the other hand, the fastest descriptor is that of Hu then that of Fourier-Mellin then that of pseudo Zernike then that of Legendre finally then that IAFMT then PZIM followed by Krawtchouk.

Competing Interests

Authors have declared that no competing interests exist.

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